Cooperative Target Enclosing Control of Multiple Mobile Robots Subject to Input Disturbances

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***Abstract*—This paper investigates the cooperative target enclosing of multiple unicycle-type mobile robots subject to input disturbances. The objective is to make all robots orbit around a given stationary target, and maintain evenly spaced along a common circle. The network of the multi-robot systems is set in a cyclic pursuit manner. A dynamic control law is developed for the cooperative target enclosing of the multi-robot systems, while tackling the heterogeneous input disturbances generated by linear exogenous systems. The proposed control law requires each robot to use the relative displacement measurements with respect to the target and its neighbors. It is shown that global asymptotic stability of the closed-loop multi-robot systems can be guaranteed in the presence of a large class of input disturbance signals. Finally, simulation results illustrate the effectiveness of our approach.**

***Index Terms*—Cooperative control, formation control, mobile robots, multi-agent systems, target enclosing, unicycles.**

I. Introduction

Since the last decade, formation control of multiple mobile robots has attracted much research interest [1], such as the autonomous surface vehicles [2], the wheeled mobile robots [3], the unmanned underwater vehicles [4] and the unmanned helicopters [5]. In particular, many works focus on the cooperative target enclosing which usually requires a team of mobile robots to orbit around a target and maintain evenly spaced along a common circle. This robotic formation has wide application prospects in monitoring or rescuing a target, for example, the ocean sampling [6].

In fact, control of mobile robotic system is a long-term hot research topic, for instance [3, 7–9]. As the unicycle model is generally used to model the simplified model of the wheeled mobile robot and the unmanned aerial vehicle [10]. Significant effort was made to the cooperative target enclosing control problem of unicycle-type mobile robots. For robots with identical linear velocity, a gradient control law based on potential function was developed for multi-robot systems with a fully connected network in [11], and an extension to a balanced network was presented in [12]. In [13], the sensory limitation in visibility of each robot was taken into account. In [14], a hybrid switching control law was developed based on the distance between each robot and target. In [15], it was shown that each robot is able to encircle the target by only using the bearing angle with respect to the target. In [16], robots were made to form an evenly spaced pattern and the size of a disk-like target was additionally considered. In [17, 18], the communication range of each robot was considered, and the results, as well as those for the cyclic pursuit problem, for instance [19, 20] can be applied to target enclosing if there is one specified robot orbiting around the target. In [21], a dynamic control law was proposed for ring-networked robots with velocity constraints. For multi-robot systems with acceleration inputs and general network topology, a control law based on a set reduction theorem was deigned in [22], and it was shown in [23] that global asymptotical stability of the closed-loop system can be guaranteed. Besides, to achieve nonidentical orbits around the target, nonidentical fixed linear velocities were set in [24, 25], and nonidentical control laws were used in [26].

However, all aforementioned works were developed for the unicycle-type mobile robots without any disturbances. These results may not be effective in the presence of input disturbances. For the trajectory tracking control problem of one single mobile robot, bounded kinematic disturbances violating the constraint of nonholonomic pure rolling and non-slipping were addressed in [27]. A comprehensive study on modeling and control of a mobile robot with the disturbances modelling the wheel skidding and slipping was given in [28]. For multiple mobile robots, in [29], the input additive disturbances in the linearized tracking error kinematics was considered, and the leader-follower formation tracking control problem was solved with the extended state observer-based distributed model predictive control approach. As pointed out in [30], the adaptive robust integral of the sign of the error (RISE) feedback can be taken as a robustifying mechanism to compensate for additive disturbances. This method has been applied to the electromechanical servo system [30] and the leader-follower formation control of mobile robots using backstepping and a neural network [31]. Some recent robust control and filter approach also addressed the uncertainty in robotic systems. In [32], the tracking control problem of a class of nonlinear systems subject to mismatched uncertainties was

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investigated and the proposed scheme was applied to a robotic manipulator. In [33] and [34], the Huber based method was used in robust filter design to achieve the attitude estimation of small satellite.

In most related existing works, the input disturbances of mobile robots were assumed as the outputs of linear exogenous systems. In the case where the exo-systems satisfy a certain condition such that the input disturbance signals will not diverge, the consensus control problem was studied in [35], and the rendezvous control and the tracking control problems were solved respectively in [36]. In the case where the system matrices of the exo-systems are skew-symmetric, equivalently input disturbances are harmonic and constant signals, internal-model-based controllers were proposed for the formation control problem of mobile robots with rigid body dynamics in [37], and the consensus control problem of unicycle-type mobile robots in [38]. For more general disturbance signals, another condition on the exo-systems was considered in [39] where the target enclosing and the trajectory tracking control problems of one mobile robot were solved respectively. Considering a class of input disturbances with known bounds, the sliding mode control technique was employed to achieve the consensus for mobile robots with velocity inputs in [40] and with acceleration inputs in [41], which may lead to chattering in the headings angles of robots.

To the best of our knowledge, there is no open existing result addressing the cooperative target enclosing control problem of multiple unicycle-type mobile robots with input disturbances. To solve this problem, we study the mobile robots in cyclic pursuit, and take into account heterogeneous input disturbances in the kinematic unicycle model. A dynamic control law is proposed such that all controlled mobile robots are able to not only orbit around a target in a common circle but also maintain evenly spaced along the circle.

The major contribution is that the global asymptotical stability of the closed-loop multi-robot systems subject to input disturbances can be guaranteed. In the aforementioned existing results on the cooperative target enclosing control problem, no input disturbance was considered. Our result achieves full rejection to a large class of input disturbances considered in [39]. Note that target enclosing for only one robot was studied in [39]. The concerned disturbances are the linear combination of a finite number of step signals and sinusoidal signals, which includes the harmonic and constant disturbance signals in [37, 38] as a special case, and describe a larger class of disturbance signals than than in [35, 36]. Our proposed control law does not lead to any chattering, while the control law in [40] may result in some. Note that the cooperative target enclosing of multiple robots was not studied in [35, 36, 40].

The remainder is organized as follows. In Section II, the problem formulation is presented. In Section III, the proposed control law is given, which is followed by the main theorem and its proof, and some discussions. Section IV shows the simulation results, and Section V draws the conclusion.

Notations: The norm $\|\boldsymbol{x}\|$ of vector $\boldsymbol{x} = [x_1, ..., x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is defined as $\|\boldsymbol{x}\| = \sqrt{\sum_{i=1}^n |x_i|^2}$. The vectors $\mathbf{0}$ and $\mathbf{1}$ denote $\mathbf{0} = [0, ..., 0]^{\mathrm{T}}$ and $\mathbf{1} = [1, ..., 1]^{\mathrm{T}}$. For a matrix A, $A > (\geq)\ 0$ and $A < (\leq)\ 0$ mean A is positive definite (semi-definite) and negative definite (semi-definite) respectively.

II. Problem Formulation

Consider $N (N \geq 2)$ unicycle-type mobile robots subject to heterogeneous input disturbances. The kinematics of each robot i, $i = 1, ..., N$, is described by

$$\begin{aligned} \dot{x}_i &= (v_i + \rho_i) \cos\theta_i, \\ \dot{y}_i &= (v_i + \rho_i) \sin\theta_i, \\ \dot{\theta}_i &= \omega_i + \varrho_i, \end{aligned} \tag{1}$$

where $\boldsymbol{p}_i := [x_i\ y_i]^{\mathrm{T}} \in \mathbb{R}^2$ and $\theta_i \in \mathbb{R}$ are the position and heading angle of robot i in the inertial frame respectively, see Fig. 1(a). $v_i \in \mathbb{R}$ and $\omega_i \in \mathbb{R}$ are the designed linear velocity and angular velocity respectively and they are the control inputs of system (1). $[\rho_i\ \varrho_i]^{\mathrm{T}} \in \mathbb{R}^2$ is the input disturbance which can be written as the output of an exogenous system

$$\dot{\boldsymbol{z}}_i = S_i \boldsymbol{z}_i,\ \rho_i = \boldsymbol{b}_i^{\mathrm{T}} \boldsymbol{z}_i,\ \varrho_i = \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{z}_i, \tag{2}$$

where $\boldsymbol{z}_i \in \mathbb{R}^{m_i}$ is the state the exo-system (2), $\boldsymbol{b}_i$, $\boldsymbol{c}_i \in \mathbb{R}^{m_i}$ are constant vectors, and $S_i \in \mathbb{R}^{m_i \times m_i}$ is the system matrix. In this paper, all exo-systems (2) satisfy the following assumption:

Assumption 1: The exo-system (2) is marginally stable, i.e., the eigenvalues of S_i have non-positive real part and those eigenvalues with zero real part are semi-simple. ∎

The objective of the *cooperative target enclosing control problem* is to design $[v_i\ \omega_i]^{\mathrm{T}}$, such that all controlled robots are able to not only orbit around a given target located at $\boldsymbol{p}_0 := [x_0\ y_0]^{\mathrm{T}}$ counterclockwise with a given radius r, but also maintain evenly spaced along the common circle.

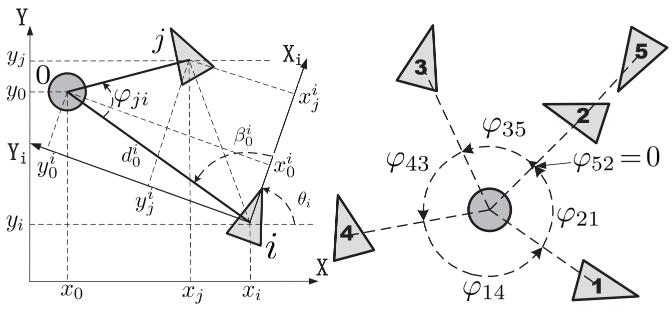
The network for the multi-robot systems is modeled by a graph $\mathcal{G} = \{\mathcal{O}, \mathcal{E}\}$, where $\mathcal{O} = \{1, ..., N\}$ is a finite set of nodes representing N mobile robots, and $\mathcal{E} \subseteq \{(j, i) : j \neq i, i, j \in \mathcal{O}\}$ is a set of edges containing directed edges from node j to node i. The information of robot j is available to robot i if $(j, i) \in \mathcal{E}$, $j \neq i$. Node j is called the neighbor of node i if $(j, i) \in \mathcal{E}$, and all neighbors of node i locate in a set $\mathcal{N}_i \subseteq \mathcal{O}$.

Denote $\boldsymbol{p}_k^i = [x_k^i\ y_k^i]^{\mathrm{T}}$, $k \in \{0, j\}$, $j \in \mathcal{N}_i$, as the coordinates of the target and the neighbors of robot i measured in the Frenet-Serret frame of robot i respectively, i.e.,

$$\boldsymbol{p}_k^i = R(\theta_i)(\boldsymbol{p}_k - \boldsymbol{p}_i),\ R(\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) \\ -\sin(\cdot) & \cos(\cdot) \end{bmatrix}. \tag{3}$$

For each robot i, $\boldsymbol{p}_k^i$ can be obtained by directly measuring the relative distances d_k^i and the bearing angles β_k^i, see Fig. 1(a) for illustration. Define $\varphi_{ji} = \angle j0i \in [-\pi, \pi)$, $i, j \in \mathcal{O}$, as the separation angle between robots j and i, see Fig. 1(a). Note that if $\boldsymbol{p}_0^i$ and $\boldsymbol{p}_j^i$, $j \in \mathcal{N}_i$, are obtained by robot i, then φ_{ji} can be directly calculated in the triangle $\triangle j0i$.

Similar to [42, 43], the network topology of the multi-robot systems is set in a cyclic pursuit manner, and is determined by the relative displacements of mobile robots with respect to the target. As in [42, 43], all mobile robots are dispersed in a counterclockwise star pattern with respect to the target at the initial time, as shown in Fig. 1(b), i.e., the initial positions of the target and all mobile robots satisfy:



(a) Variables (b) Cyclic pursuit manner

Fig. 1. Illustration of the variables and the cyclic pursuit manner.

Assumption 2: All mobile robots and the target have distinct initial positions, i.e., $\boldsymbol{p}_i(t_0) \neq \boldsymbol{p}_j(t_0) \neq \boldsymbol{p}_0(t_0),\ \forall i \neq j$. ■

Then, the group of mobile robots are sorted based on the real-time counterclockwise radial order around the target. In the case where $\varphi_{ji}(t) = 0,\ d_0^j(t) > d_0^i(t) > 0$, robot i is sorted before robot j in the counterclockwise radial order. For example, the counterclockwise radial order shown in Fig. 1(b) is $1 \to 2 \to 5 \to 3 \to 4 \to 1$. At each time instant, each robot i is required to obtain φ_{ji} with respect to two robots sorted before robot i and φ_{ij} with respect to two robots sorted after robot i in the counterclockwise radial order. Denote these four neighbors of robot i as robots i++, i+, i−, and i−− respectively. That is, the following assumption is made.

Assumption 3: Each mobile robot i is able to find its four neighbors i++, i+, i−, and i−− in the network, i.e., $\mathcal{N}_i(t) = \{i{+}{+}, i{+}, i{-}, i{-}{-}\},\ \forall i \in \mathcal{O},\ \forall t \geq t_0$. ■

Finally, the *cooperative target enclosing control problem* are formally defined as follows.

Problem 1: Given a target located at $\boldsymbol{p}_0 = [x_0\ y_0]^{\mathrm{T}}$ and a radius r, for a group of mobile robot (1) subject to heterogeneous disturbances generated by the exo-systems (2), with any initial states $[\boldsymbol{p}_i^{\mathrm{T}}(t_0)\ \theta_i(t_0)]^{\mathrm{T}} \in \mathbb{R}^3,\ \forall t_0 \geq 0$, find a dynamic control law in the form of

$$[v_i\ \omega_i]^{\mathrm{T}} = \boldsymbol{\vartheta}(\boldsymbol{p}_0^i, \boldsymbol{p}_j^i, \boldsymbol{\xi}_i, r), \tag{4}$$

$$\dot{\boldsymbol{\xi}}_i = \boldsymbol{\varsigma}(\boldsymbol{p}_0^i, \boldsymbol{p}_j^i, \boldsymbol{\xi}_i, r),\ j \in \mathcal{N}_i(t), \tag{5}$$

such that each $\boldsymbol{p}_i(t)$ is bounded for all $t \geq t_0$ and for all $i \in \mathcal{O}$,

$$\lim_{t\to\infty} (\boldsymbol{p}_i(t) - \boldsymbol{p}_0) = r \begin{bmatrix} \sin\theta_i(t) \\ -\cos\theta_i(t) \end{bmatrix},\ \dot{\theta}_i > 0, \tag{6}$$

$$\lim_{t\to\infty} \|\boldsymbol{p}_i(t) - \boldsymbol{p}_{i^+}(t)\| = \lim_{t\to\infty} \|\boldsymbol{p}_i(t) - \boldsymbol{p}_{i^-}(t)\|, \tag{7}$$

where $\boldsymbol{\xi}_i$ is some internal state to be designed, $\boldsymbol{\vartheta}(\cdot)$ and $\boldsymbol{\varsigma}(\cdot)$ are sufficiently smooth functions, and i+ and i− denote the two nearest neighbors of robot i in the initial counterclockwise and clockwise radial orders around the target respectively. ■

The objective (6) defines the target enclosing of each robot, while the objective (7) defines the evenly spaced formation of all robots. Note that (7) can be achieved if all separation angles $\varphi_{(i+)i}$ between each robot i and its neighbour i+ approach the same value.

Remark 2.1: Under Assumption 1, the input disturbance signal $[\rho_i\ \varrho_i]^{\mathrm{T}}$ can be written in the form of

$$\rho_i(t) = \alpha_i + \textstyle\sum_{j=1}^{n_i} \kappa_{ij} \sin(\gamma_{ij} t + \psi_{ij}), \tag{8}$$

$$\varrho_i(t) = \alpha_i' + \textstyle\sum_{j=1}^{m_i} \kappa_{ij}' \sin(\gamma_{ij}' t + \psi_{ij}'), \tag{9}$$

with unknown amplitudes α_i, α_i', κ_{ij}, and κ_{ij}', and unknown phases ψ_{ij}, and ψ_{ij}'. The disturbance signals (8)–(9) are the linear combination of a finite number of step signals and sinusoidal signals. Since a periodic signal can be represented as a sum of sinusoids by Fourier series expansion, a large class of persistent disturbances can be described by (8) and (9). Note that this theoretical framework accommodates the periodical disturbances, which may be suitable for the mobile robots in some practical scenarios, since the input disturbances in the velocity channels of mobile robots are often caused by the systematic errors. For instance, the disturbances in the spinning wheels of a differential drive wheeled mobile robot are often caused by some systematic malfunction of engine or low-level actuator, such as motor offset, friction of mechanical gear, and wear and tear of wheels. Moreover, under Assumption 1, the disturbance signals are more general than that considered in [35–38]. For instance, the concerned disturbance signals include the harmonic and constant disturbance signals in [37, 38] as a special case. ■

Remark 2.2: The cooperative target enclosing can also be achieved by the leader-follower formation tracking control, for instance [3, 29, 36, 41], by setting a virtual leader with state $[x_r(t)\ y_r(t)\ \theta_r(t)]^{\mathrm{T}}$ on an orbit trajectory around the given target with radius r and linear velocity v_0. However, this strategy requires the mobile robot to obtain the actual or relative heading angle measurements with respect to the leader robot and its neighbors, i.e., θ_i and θ_r, or $\theta_r - \theta_i$. While these measurements are not needed in this paper, which makes the implementation much easier in practice. In fact, encircling the target only requires $[d_0^i\ \beta_0^i]^{\mathrm{T}}$, i.e., the relative distance and bearing angles, to converge to $[r\ -\frac{\pi}{2}]^{\mathrm{T}}$, and forming the evenly spaced formation only needs each septation angle $\varphi_{(i+)i}$ in each $\triangle(i+)0i$ to converge to the same value. Thus, the relative heading angles have no direct contribute to both objectives. ■

III. Main Results

In this section, we first reformulate objectives (6)–(7) to facilitate the control law design and propose a dynamic control law to solve the *cooperative target enclosing control problem*. Then, the main theorem with the proof is presented. Finally, some discussions along with two corollaries are given.

A. Control law design

First, we define a tracking error $\boldsymbol{e}_i := [\boldsymbol{e}_{xi}\ \boldsymbol{e}_{yi}]^{\mathrm{T}}$ as

$$\boldsymbol{e}_i = R(\theta_i)(\boldsymbol{p}_0 - \boldsymbol{p}_i + r \begin{bmatrix} \sin\theta_i \\ -\cos\theta_i \end{bmatrix}) = \boldsymbol{p}_0^i - \begin{bmatrix} 0 \\ r \end{bmatrix}. \tag{10}$$

Then, we have the following error dynamics:

$$\dot{\boldsymbol{e}}_i = (\omega_i + \varrho_i) A \boldsymbol{e}_i - (v_i - \omega_i r + \rho_i - \varrho_i r) \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{11}$$

where $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Thus, to make all mobile robots orbit around the target counterclockwise with the given radius r, i.e., to achieve (6), it suffices to show that

$$\lim_{t\to\infty} \boldsymbol{e}_i(t) = \boldsymbol{0}. \tag{12}$$

Next, the upper right-hand time derivative of the separation angle $\varphi_{(i+)i}$ is

$$\mathrm{D}^+\varphi_{(i+)i} = \frac{(v_{i+}+\rho_{i+})\sin\beta_0^{i+}}{d_0^{i+}} - \frac{(v_i+\rho_i)\sin\beta_0^{i}}{d_0^{i}},\ i\in\mathcal{O}, \tag{13}$$

where $\beta_0^i \in [-\pi,\pi)$ is the bearing angle of robot i with respect to the target satisfying $d_0^i[\cos\beta_0^i\ \ \sin\beta_0^i]^{\mathrm{T}} = \boldsymbol{p}_0^i$; see Fig. 1(a). Thus, to make all mobile robots maintain evenly spaced along the common circle, i.e., to achieve (7), it suffices to show that

$$\lim_{t\to\infty} \varphi_{(i+)i}(t) = \frac{2\pi}{N},\ \forall i\in\mathcal{O}. \tag{14}$$

In what follows, we design a dynmaic control law in the form of (4)–(5), such that the objectives (12) and (14) are achieved. To this end, we introduce the internal states as

$$\hat{\boldsymbol{e}}_i := [\hat{e}_{xi}\ \hat{e}_{yi}]^{\mathrm{T}} \in \mathbb{R}^2,\ \hat{\boldsymbol{z}}_i \in \mathbb{R}^{m_i},\ \hat{\boldsymbol{u}}_i \in \mathbb{R}^{m_i}, \tag{15}$$

and define the following internal state:

$$\hat{\rho}_i = \boldsymbol{b}_i^{\mathrm{T}}\hat{\boldsymbol{z}}_i,\ \hat{\varrho}_i = \boldsymbol{c}_i^{\mathrm{T}}\hat{\boldsymbol{z}}_i,\ \hat{\sigma}_i = \boldsymbol{b}_i^{\mathrm{T}}\hat{\boldsymbol{u}}_i. \tag{16}$$

The initial states $[\hat{\boldsymbol{e}}_i^{\mathrm{T}}(t_0)\ \hat{\boldsymbol{z}}_i^{\mathrm{T}}(t_0)\ \hat{\boldsymbol{u}}_i^{\mathrm{T}}(t_0)]^{\mathrm{T}}$ can be arbitrarily chosen in $\mathbb{R}^2\times\mathbb{R}^{m_i}\times\mathbb{R}^{m_i}$.

We then propose a dynamic control law as follows.

$$v_i = v_0 - \hat{\rho}_i - \hat{\sigma}_i + k_v\tanh(\varphi_{(i+)i} - \varphi_{i(i-)}), \tag{17}$$

$$\omega_i = \frac{v_i+\hat{\rho}_i}{r} - \hat{\varrho}_i - \frac{k_\omega \hat{e}_{xi}}{r\sqrt{1+\hat{e}_{xi}^2}}, \tag{18}$$

$$\dot{\hat{\boldsymbol{e}}}_i = (\omega_i+\hat{\varrho}_i)A\hat{\boldsymbol{e}}_i + K_i(\boldsymbol{e}_i - \hat{\boldsymbol{e}}_i) + \frac{k_\omega(\hat{e}_{xi}-e_{xi})}{\sqrt{1+\hat{e}_{xi}^2}}\begin{bmatrix}1\\0\end{bmatrix}, \tag{19}$$

$$\begin{aligned}\dot{\hat{\boldsymbol{z}}}_i = {} & S_i\hat{\boldsymbol{z}}_i - P_i^{-1}([\boldsymbol{b}_i\ \boldsymbol{c}_i]\begin{bmatrix}1\\-r\end{bmatrix}(\hat{e}_{xi}-2e_{xi}) \\ & - \boldsymbol{c}_i(\boldsymbol{e}_i-\hat{\boldsymbol{e}}_i)^{\mathrm{T}}A\boldsymbol{e}_i),\end{aligned} \tag{20}$$

$$\begin{aligned}\dot{\hat{\boldsymbol{u}}}_i = {} & S_i\hat{\boldsymbol{u}}_i - \frac{P_i^{-1}\boldsymbol{b}_i}{r}(2\tanh(\varphi_{(i+)i}-\varphi_{i(i-)}) \\ & - \tanh(\varphi_{(i++)i} - 2\varphi_{(i+)i}) - \tanh(2\varphi_{i(i-)} - \varphi_{i(i--)})),\end{aligned} \tag{21}$$

where v_0 is a given positive constant, parameters k_v and k_ω are any positive constants, $K_i \in \mathbb{R}^{2\times 2}$ is any matrix satisfying $K_i + K_i^{\mathrm{T}} > 0$, and $P_i \in \mathbb{R}^{m\times m}$ is a positive definite matrix satisfying:

$$P_iS_i + S_i^{\mathrm{T}}P_i \le 0. \tag{22}$$

Note that there exists a matrix $P_i > 0$ satisfying inequality (22) if and only if Assumption 1 is satisfied; see [44]. As $\boldsymbol{e}_i$ can be obtained by measuring $\boldsymbol{p}_0^i$ according to (10) and φ_{ji} can be calculated with $\boldsymbol{p}_0^i(t)$ and $\boldsymbol{p}_j^i(t)$, $j\in\mathcal{N}_i$, in the triangle $\triangle j0i$, the proposed control law (17)–(21) is in the form of (4)–(5). In particular, controller ω_i in (18) is designed for achieving the convergence to circular motion around the target. Controller v_i in (17) makes the robots simultaneously converge to the evenly spaced formation and function $\tanh(\cdot)$ is set so as to limit the bound of the error $\varphi_{(i+)i} - \varphi_{i(i-)}$ by k_v. While (19)–(21) are designed as the update law for the internal states, so as to handle the uncertainties brought by the input disturbances.

B. Main theorem

The main result of this paper is presented as follows.

Theorem 1: Under Assumptions 1–3, the *cooperative target enclosing control problem* defined in Problem 1 is solved by control law (17)–(21), if each pair $(\boldsymbol{b}_i^{\mathrm{T}}, S_i)$, $i\in\mathcal{O}$, is observable. ■

Proof: The proof of Theorem 1 is established with the so-called set reduction theorem [45, Proposition 14], and is proceeded in the following two steps.

Denote $\boldsymbol{e} = [\boldsymbol{e}_1^{\mathrm{T}}, \boldsymbol{e}_{1+}^{\mathrm{T}}, ..., \boldsymbol{e}_{1-}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2N}$, $\hat{\boldsymbol{e}} = [\hat{\boldsymbol{e}}_1^{\mathrm{T}}, \hat{\boldsymbol{e}}_{1+}^{\mathrm{T}}, ..., \hat{\boldsymbol{e}}_{1-}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2N}$, and $\boldsymbol{\varphi} = [\varphi_{(1+)1},\ \varphi_{(1++)(1+)}, ..., \varphi_{(1-)1}]^{\mathrm{T}} \in [-\pi,\pi)^N$. Note that $\boldsymbol{\varphi}$ is determined by $\boldsymbol{e}$ and is a function of $\boldsymbol{e}$.

In the first step, we prove that all mobile robots converge to orbiting along the common circle with the center $\boldsymbol{p}_0$ and the radius r. That is, the following proposition can be obtained.

Proposition 1: Consider N systems (11) under Assumptions 1–3. Control law (17)–(21) guarantees that the trajectory $[\boldsymbol{e}^{\mathrm{T}}(t)\ \hat{\boldsymbol{e}}^{\mathrm{T}}(t)]^{\mathrm{T}}$ is globally uniformly bounded, and that if each pair $(\boldsymbol{b}_i^{\mathrm{T}}, S_i)$, $i\in\mathcal{O}$, is observable, set Γ_1 defined as

$$\Gamma_1 = \{[\boldsymbol{e}^{\mathrm{T}}\ \hat{\boldsymbol{e}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2N}\times\mathbb{R}^{2N} | \hat{\boldsymbol{e}} = \boldsymbol{e} = \boldsymbol{0}\}, \tag{23}$$

is globally asymptotically stable relative to $\mathbb{R}^{2N}\times\mathbb{R}^{2N}$. ■

In the second step, we show that the mobile robots orbiting along the prescribed common circle, converge to an evenly spaced formation, i.e., the following proposition is obtained.

Proposition 2: Consider N systems (13) under Assumptions 1–3. Control law (17)–(21) guarantees that set Γ_2 defined as

$$\Gamma_2 = \{[\boldsymbol{e}^{\mathrm{T}}\ \hat{\boldsymbol{e}}^{\mathrm{T}}]^{\mathrm{T}} \in \Gamma_1 : \boldsymbol{\varphi}(\boldsymbol{e}) = \frac{2\pi}{N}\boldsymbol{1}\}, \tag{24}$$

is globally asymptotically stable relative to Γ_1. ■

Finally, with Propositions 1 and 2, Theorem 1 can be proved by directly using the so-called set reduction theorem [45, Proposition 14]. The detailed proofs of Propositions 1 and 2 are presented below.

1) Proof of Proposition 1: To prove Proposition 1, the following lemma is needed.

Lemma 1: Consider a linear marginally stable system $\dot{\boldsymbol{\zeta}} = Q\boldsymbol{\zeta}$, $\eta = \boldsymbol{q}^{\mathrm{T}}\boldsymbol{\zeta}$, with state $\boldsymbol{\zeta}\in\mathbb{R}^m$, constant vector $\boldsymbol{q}\in\mathbb{R}^m$, and matrix $Q\in\mathbb{R}^{m\times m}$. If the pair $(\boldsymbol{q}^{\mathrm{T}}, Q)$ is observable and the initial state satisfies $Q\boldsymbol{\zeta}(t_0) \neq \boldsymbol{0}$, then $\lim_{t\to\infty}\eta(t)$ does not exist. ■

Proof: We use a contradiction argument to prove Lemma 1. Suppose that there exists an η_0 such that $\lim_{t\to\infty}\eta(t) = \eta_0$. Since the concerned system is marginally stable, then $\eta(t) \equiv \eta_0$. Next, we write $\left[\eta;\dot{\eta};...;\eta^{(n-1)}\right] = \left[\boldsymbol{q}^{\mathrm{T}};\boldsymbol{q}^{\mathrm{T}}Q;...;\boldsymbol{q}^{\mathrm{T}}Q^{n-1}\right]\boldsymbol{\zeta}(t) = [\eta_0;0;...;0]$. As (b', S) is observable, then $rank(\left[\boldsymbol{q}^{\mathrm{T}};\boldsymbol{q}^{\mathrm{T}}Q;...;\boldsymbol{q}^{\mathrm{T}}Q^{n-1}\right]) = m$, which implies that equation $\left[\boldsymbol{q}^{\mathrm{T}};\boldsymbol{q}^{\mathrm{T}}Q;...;\boldsymbol{q}^{\mathrm{T}}Q^{n-1}\right]\boldsymbol{\zeta}(t) = [\eta_0;0;...;0]$

has one unique solution $\overline{\boldsymbol{\zeta}}$. Thus, we have $\boldsymbol{\zeta}(t) \equiv \overline{\boldsymbol{\zeta}}$. In this case, $\dot{\boldsymbol{\zeta}}(t) = Q\boldsymbol{\zeta}(t) = Q\overline{\boldsymbol{\zeta}} = \mathbf{0}$ and then $Q\boldsymbol{\zeta}(t_0) = \mathbf{0}$, which contradicts to $Q\boldsymbol{\zeta}(t_0) \neq \mathbf{0}$. Hence, $\lim\limits_{t\to\infty} \eta(t)$ does not exist. ■

Then, the proof of Proposition 1 is given as follows.

Define $\tilde{\boldsymbol{e}}_i = [\tilde{e}_i^x \ \tilde{e}_i^y]^{\mathrm{T}}$, $\tilde{\boldsymbol{z}}_i$, $\tilde{\rho}_i$ and $\tilde{\varrho}_i$ as

$$\tilde{\boldsymbol{e}}_i = \hat{\boldsymbol{e}}_i - \boldsymbol{e}_i,\ \tilde{\boldsymbol{z}}_i = \hat{\boldsymbol{z}}_i - \boldsymbol{z}_i,\ \tilde{\rho}_i = \boldsymbol{b}_i^{\mathrm{T}}\tilde{\boldsymbol{z}}_i,\ \tilde{\varrho}_i = \boldsymbol{c}_i^{\mathrm{T}}\tilde{\boldsymbol{z}}_i, \tag{25}$$

and we have $\tilde{\rho}_i = \hat{\rho}_i - \rho_i$ and $\tilde{\varrho}_i = \hat{\varrho}_i - \varrho_i$. Then, the augmented closed-loop system consisting of (11), and control law (17)–(20) is written as

$$\dot{\boldsymbol{e}}_i = (\omega_i + \varrho_i)A\boldsymbol{e}_i - (\tilde{\varrho}_i r - \tilde{\rho}_i + \frac{k_\omega(\tilde{e}_{xi} + e_{xi})}{\sqrt{1+\hat{e}_{xi}^2}})\begin{bmatrix}1\\0\end{bmatrix}, \tag{26}$$

$$\begin{aligned}\dot{\tilde{\boldsymbol{e}}}_i &= (\omega_i + \hat{\varrho}_i)A\tilde{\boldsymbol{e}}_i + \tilde{\varrho}_i A\boldsymbol{e}_i - K_i\tilde{\boldsymbol{e}}_i \\ &\quad + (\tilde{\varrho}_i r - \tilde{\rho}_i + \frac{k_\omega e_{xi}}{\sqrt{1+\hat{e}_{xi}^2}})\begin{bmatrix}1\\0\end{bmatrix},\end{aligned} \tag{27}$$

$$\dot{\tilde{\boldsymbol{z}}}_i = S_i\tilde{\boldsymbol{z}}_i - P_i^{-1}(\boldsymbol{c}_i\tilde{\boldsymbol{e}}_i^{\mathrm{T}}A\boldsymbol{e}_i + [\boldsymbol{b}_i\ \boldsymbol{c}_i]\begin{bmatrix}1\\-r\end{bmatrix}(\tilde{e}_{xi} - e_{xi})). \tag{28}$$

Consider a positive definite and decrescent Lyapunov function candidate $V_i(t, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i) : \mathbb{R}_{\geq 0} \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^{m_i} \to \mathbb{R}$ as $V_i(t, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i) = \frac{1}{2}\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i + \frac{1}{2}\tilde{\boldsymbol{e}}_i^{\mathrm{T}}\tilde{\boldsymbol{e}}_i + \frac{1}{2}\tilde{\boldsymbol{z}}_i^{\mathrm{T}}P\tilde{\boldsymbol{z}}_i$. Taking its time derivative along the trajectories of system (26)–(28) leads to

$$\begin{aligned}\dot{V}_i &= \frac{\omega_i + \varrho_i}{2}\boldsymbol{e}_i^{\mathrm{T}}(A + A^{\mathrm{T}})\boldsymbol{e}_i - e_{xi}(\tilde{\varrho}_i r - \tilde{\rho}_i) + \tilde{\varrho}_i\tilde{\boldsymbol{e}}_i^{\mathrm{T}}A\boldsymbol{e}_i \\ &\quad + \frac{\omega_i + \varrho_i}{2}\tilde{\boldsymbol{e}}_i^{\mathrm{T}}(A + A^{\mathrm{T}})\tilde{\boldsymbol{e}}_i - \frac{1}{2}\tilde{\boldsymbol{e}}_i^{\mathrm{T}}(K_i + K_i^{\mathrm{T}})\tilde{\boldsymbol{e}}_i \\ &\quad + \frac{k_\omega e_{xi}\tilde{e}_{xi}}{\sqrt{1+\hat{e}_{xi}^2}} + \tilde{e}_{xi}(\tilde{\varrho}_i r - \tilde{\rho}_i) - (\tilde{\varrho}_i r - \tilde{\rho}_i)(\tilde{e}_{xi} - e_{xi}) \\ &\quad + \frac{1}{2}\tilde{\boldsymbol{z}}_i^{\mathrm{T}}(P_iS_i + S_i^{\mathrm{T}}P_i)\tilde{\boldsymbol{z}}_i - \tilde{\varrho}_i\tilde{\boldsymbol{e}}_i^{\mathrm{T}}A\boldsymbol{e}_i - \frac{k_\omega(e_{xi}\tilde{e}_{xi} + e_{xi}^2)}{\sqrt{1+\hat{e}_{xi}^2}} \\ &\leq -\frac{k_\omega e_{xi}^2}{\sqrt{1+\hat{e}_{xi}^2}} - \frac{1}{2}\tilde{\boldsymbol{e}}_i^{\mathrm{T}}(K_i + K_i^{\mathrm{T}})\tilde{\boldsymbol{e}}_i \leq 0,\end{aligned} \tag{29}$$

since $A + A^{\mathrm{T}} = 0$, $P_iS_i + S_i^{\mathrm{T}}P_i \leq 0$, and $K_i + K_i^{\mathrm{T}} > 0$. It follows from (29) that the closed-loop system (26)–(28) is globally uniformly stable and $\boldsymbol{e}_i$, $\tilde{\boldsymbol{e}}_i$ and $\tilde{\boldsymbol{z}}_i$ are bounded in t. Since $V_i(t, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i)$ is nonincreasing in t and bounded, $\lim\limits_{t\to\infty} \int_{t_0}^{t} \dot{V}_i(\tau, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i)\mathrm{d}\tau$ exists and is finite. By (26)–(28), $\dot{\boldsymbol{e}}_i$, $\dot{\tilde{\boldsymbol{e}}}_i$, and $\dot{\tilde{\boldsymbol{z}}}_i$ are also bounded in t. Note $\dot{V}_i(t, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i) = \frac{1}{2}\tilde{\boldsymbol{z}}_i^{\mathrm{T}}(P_iS_i + S_i^{\mathrm{T}}P_i)\tilde{\boldsymbol{z}}_i - \frac{k_\omega e_{xi}^2}{\sqrt{1+\hat{e}_{xi}^2}} - \frac{1}{2}\tilde{\boldsymbol{e}}_i^{\mathrm{T}}(K_i + K_i^{\mathrm{T}})\tilde{\boldsymbol{e}}_i$ in (29), it follows that $\ddot{V}_i(t, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i)$ is bounded in t. Then, $\dot{V}_i(t, \boldsymbol{e}_i, \tilde{\boldsymbol{e}}_i, \tilde{\boldsymbol{z}}_i)$ is uniformly continuous in t. Using Barbalat's Lemma, we obtain

$$\lim_{t\to\infty} e_{xi}(t) = 0,\ \ \lim_{t\to\infty} \tilde{\boldsymbol{e}}_i(t) = \mathbf{0}. \tag{30}$$

Next, the extended Barbalat's Lemma [46, Lemma A.14] is used to prove $\lim\limits_{t\to\infty} e_{yi}(t) = 0$.

According to (19), the time derivative of $\hat{e}_{xi}$ is written as $\dot{\hat{e}}_{xi} = h_{i1} + h_{i2}$ with $h_{i1} = (\omega_i + \hat{\varrho}_i)e_{yi} = \frac{\bar{v}_i}{r}e_{yi}$ and $h_{i2} = (\omega_i + \hat{\varrho}_i)\tilde{e}_{yi} - k_\omega\tilde{e}_{xi} - [1\ \ 0]K_i\tilde{\boldsymbol{e}}_i$, where

$$\bar{v}_i = v_0 - \hat{\sigma}_i + k_v\tanh(\varphi_{(i+)i} - \varphi_{i(i-)}). \tag{31}$$

Using (30), we have $\lim\limits_{t\to\infty} h_{i2}(t) = 0$. Since $\boldsymbol{e}_i$ is bounded, it follows from (26) that $\dot{h}_1(t)$ exists and is also bounded, which implies $h_{i1}(t)$ is uniformly continuous in t. Using the extended Barbalat's Lemma [46, Lemma A.14], we obtain $\lim\limits_{t\to\infty} h_{i1}(t) = 0$. Then, it follows that

$$\lim_{t\to\infty} e_{yi}(t) = 0, \tag{32}$$

provided that $\bar{v}_i(t)$ in (31) does not converge to 0 as $t \to \infty$.

Now, we prove that $\bar{v}_i(t)$ does not converge to 0 by contradiction. Suppose that $\bar{v}_i(t) \to 0$ as $t \to \infty$. Denote $\hat{\boldsymbol{u}} = [\hat{\boldsymbol{u}}_1^{\mathrm{T}}, \hat{\boldsymbol{u}}_{1^+}^{\mathrm{T}}, ..., \hat{\boldsymbol{u}}_{1^-}^{\mathrm{T}}]^{\mathrm{T}}$ and $\hat{\boldsymbol{\sigma}} = [\hat{\sigma}_1, \hat{\sigma}_{1^+}, ..., \hat{\sigma}_{1^-}]^{\mathrm{T}}$. As $\bar{v}_i(t)$ converges to 0, $\hat{\boldsymbol{u}}(t)$ converges to set $\hat{U}$ defined as

$$\hat{U} = \{\hat{\boldsymbol{u}} \in \mathbb{R}^{\sum_{j=1}^{N} m_i} | \bar{v}_i(\hat{\boldsymbol{u}}_i) = 0,\ i \in \mathcal{O}\}. \tag{33}$$

Moreover, define

$$\Delta_i = \varphi_{(i+)i} - \varphi_{i(i-)}, \tag{34}$$

and denote $\boldsymbol{\Delta} = [\Delta_1, \Delta_{1^+}, ..., \Delta_{1^-}]^{\mathrm{T}}$. It follows from (31) that when $\hat{\boldsymbol{u}} \in \hat{U}$,

$$k_v\tanh\Delta_i = -(v_0 - \hat{\sigma}_i). \tag{35}$$

Then, it follows from (21) and (35) that for all $\hat{\boldsymbol{u}} \in \hat{U}$, $\dot{\hat{\boldsymbol{u}}}$ satisfies

$$\dot{\hat{\boldsymbol{u}}} = S\hat{\boldsymbol{u}} - \frac{1}{r}P^{-1}BL(\boldsymbol{\Delta})\hat{\boldsymbol{\sigma}}, \tag{36}$$

where $S = \mathrm{diag}\{S_1, S_{1^+}, ..., S_{1^-}\}$, $P = \mathrm{diag}\{P_1, P_{1^+}, ..., P_{1^-}\}$, $B = \mathrm{diag}\{b_1, b_{1^+}, ..., b_{1^-}\}$. $L(\boldsymbol{\Delta})$ denotes the Laplacian matrix of a graph which is always a cycle.

Next, consider a Lyapunov function candidate $V(\hat{\boldsymbol{u}}) = \frac{1}{2}\hat{\boldsymbol{u}}^{\mathrm{T}}P\hat{\boldsymbol{u}}$ and take its the upper right-hand time derivative along the trajectory of system (36) yields

$$\begin{aligned}\mathrm{D}^+V(\hat{\boldsymbol{u}}) &= \frac{1}{2}\hat{\boldsymbol{u}}^{\mathrm{T}}(PS + S^{\mathrm{T}}P)\hat{\boldsymbol{u}} - \frac{1}{2r}\hat{\boldsymbol{\sigma}}^{\mathrm{T}}(L(\boldsymbol{\Delta}) + L(\boldsymbol{\Delta})^{\mathrm{T}})\hat{\boldsymbol{\sigma}} \\ &\leq -\frac{1}{2r}\hat{\boldsymbol{\sigma}}^{\mathrm{T}}(L(\boldsymbol{\Delta}) + L(\boldsymbol{\Delta})^{\mathrm{T}})\hat{\boldsymbol{\sigma}} \\ &= -\frac{1}{r}\hat{\boldsymbol{\sigma}}^{\mathrm{T}}L(\boldsymbol{\Delta})\hat{\boldsymbol{\sigma}} \leq 0,\end{aligned} \tag{37}$$

where it is noted that $L(\boldsymbol{\Delta})$ is always positive semi-definite, and $PS + S^{\mathrm{T}}P \leq 0$ by (22). Define set $U = \{\hat{\boldsymbol{u}} \in \hat{U} | \mathrm{D}^+V(\hat{\boldsymbol{u}}) = 0\}$. It can be concluded from the definition of $L(\boldsymbol{\Delta})$ that

$$U = \{\hat{\boldsymbol{u}} \in \hat{U} | b_i^{\mathrm{T}}\hat{\boldsymbol{u}}_i = \hat{\sigma}_i = \hat{\varpi},\ i \in \mathcal{O}\}, \tag{38}$$

with some variable $\hat{\varpi}$, and that for all $\hat{\boldsymbol{u}} \in \hat{U}$, $\hat{\boldsymbol{u}}$ converges to set U by the non-smooth LaSalle's invariance principle [47, Theorem 3.2, Chapter VII].

Since $\Delta_1 = \Delta_{1^+} = ... = \Delta_{1^-}$ by (35), and $\Delta_1 + \Delta_{1^+} + ... + \Delta_{1^-} \equiv 0$ by (34), then $\Delta_i = 0$, $\forall i \in \mathcal{O}$ is obtained. Thus, we have $U = \{\hat{\boldsymbol{u}} \in \hat{U} | b_i^{\mathrm{T}}\hat{\boldsymbol{u}}_i = \hat{\varpi},\ i \in \mathcal{O}\}$, and $\bar{v}_i(\hat{\boldsymbol{u}}_i) = v_0 - \hat{\varpi} = 0$, $\forall \hat{\boldsymbol{u}} \in U$. Using [48, Theorem 2.10], it follows from (35) and (36) that

$$\dot{\hat{\boldsymbol{u}}}_i = S_i\hat{\boldsymbol{u}}_i,\ \hat{\sigma}_i = \boldsymbol{b}_i^{\mathrm{T}}\hat{\boldsymbol{u}}_i,\ \forall\hat{\boldsymbol{u}} \in U. \tag{39}$$

By Lemma 1, $\hat{\sigma}_i$ does not converge to a constant, i.e., $\hat{\varpi}$ is not a constant. However, $\bar{v}_i(\hat{\boldsymbol{u}}_i) = v_0 - \hat{\varpi} = 0$ leads to $v_0 = \hat{\varpi}$, which contradicts the fact that v_0 is a positive constant. Thus, $U = \emptyset$ holds. Since for all $\hat{\boldsymbol{u}} \in \hat{U}$, $\hat{\boldsymbol{u}}$ converges to set U, then

$\hat{U} = \emptyset$ and $\bar{v}_i$ does not converge to 0 as $t \to \infty$.

Hence, the closed-loop system (26)–(28) is globally asymptotically stable, that is, Γ_1 is globally asymptotically stable relative to $\mathbb{R}^{2N} \times \mathbb{R}^{2N}$. The proof is thus completed. ■

2) Proof of Proposition 2: When $[\boldsymbol{e}^{\mathrm{T}}\ \hat{\boldsymbol{e}}^{\mathrm{T}}]^{\mathrm{T}} \in \Gamma_1$, i.e., $\boldsymbol{e} = \hat{\boldsymbol{e}} = \boldsymbol{0}$, we have $d_0^i = r$ and $\beta_0^i = \frac{\pi}{2}$, $\forall i \in \mathcal{O}$. Then, with (17), system (13) becomes

$$\mathrm{D}^+\varphi_{(i+)i} = \frac{-\tilde{\sigma}_{i+} + k_v \tanh \Delta_{i+}}{r} - \frac{-\tilde{\sigma}_i + k_v \tanh \Delta_i}{r}, \tag{40}$$

where Δ_i is defined in (34) and $\tilde{\sigma}_i$ is defined as

$$\tilde{\sigma}_i = \boldsymbol{b}_i^{\mathrm{T}} \tilde{\boldsymbol{u}}_i,\ \tilde{\boldsymbol{u}}_i = \hat{\boldsymbol{u}}_i + \tilde{\boldsymbol{z}}_i. \tag{41}$$

It follows from (28) that $\dot{\tilde{\boldsymbol{z}}}_i = S_i \tilde{\boldsymbol{z}}_i$ when $[\boldsymbol{e}^{\mathrm{T}}\ \hat{\boldsymbol{e}}^{\mathrm{T}}]^{\mathrm{T}} \in \Gamma_1$. Then, using (21) and (40), the upper right-hand time derivative of Δ_i and the time derivative of $\tilde{\boldsymbol{u}}_i$ are expressed as

$$\begin{aligned} \mathrm{D}^+\Delta_i = &-\frac{1}{r}(k_v(2\tanh\Delta_i - \tanh\Delta_{i+} - \tanh\Delta_{i-}) \\ &+ (2\tilde{\sigma}_i - \tilde{\sigma}_{i+} - \tilde{\sigma}_{i-})), \end{aligned} \tag{42}$$

$$\dot{\tilde{\boldsymbol{u}}}_i = S_i \tilde{\boldsymbol{u}}_i - \frac{P_i^{-1}\boldsymbol{b}_i}{r}(2\tanh\Delta_i - \tanh\Delta_{i+} - \tanh\Delta_{i-}), \tag{43}$$

respectively. Denote $\tilde{\boldsymbol{u}} = [\tilde{\boldsymbol{u}}_1^{\mathrm{T}}, \tilde{\boldsymbol{u}}_{1+}^{\mathrm{T}}, ..., \tilde{\boldsymbol{u}}_{1-}^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{\boldsymbol{\sigma}} = [\tilde{\sigma}_1, \tilde{\sigma}_{1+}, ..., \tilde{\sigma}_{1-}]^{\mathrm{T}}$, and $\boldsymbol{T}(\boldsymbol{\Delta}) = [\tanh\Delta_1, \tanh\Delta_{1+}, ..., \tanh\Delta_{1-}]^{\mathrm{T}}$. Then, N systems in the form of (42)–(43) are rewritten in the following compact form:

$$\mathrm{D}^+\boldsymbol{\Delta} = -\frac{k_v}{r}L(\boldsymbol{\Delta})\boldsymbol{T}(\boldsymbol{\Delta}) + \frac{1}{r}L(\boldsymbol{\Delta})\tilde{\boldsymbol{\sigma}}, \tag{44}$$

$$\dot{\tilde{\boldsymbol{u}}} = S\tilde{\boldsymbol{u}} - \frac{1}{r}P^{-1}BL(\boldsymbol{\Delta})\boldsymbol{T}(\boldsymbol{\Delta}). \tag{45}$$

Next, consider a Lyapunov function candidate $V(\boldsymbol{\Delta}, \tilde{\boldsymbol{u}}) : \mathbb{R}^N \times \mathbb{R}^{\sum_{j=1}^N m_i} \to \mathbb{R}$ as $V(\boldsymbol{\Delta}, \tilde{\boldsymbol{u}}) = \frac{1}{2}\sum_{i=1}^N \log(\cosh\Delta_i) + \frac{1}{2}\tilde{\boldsymbol{u}}^{\mathrm{T}}P\tilde{\boldsymbol{u}}$. Taking its upper right-hand time derivative along the trajectories of system (44)–(45) yields

$$\begin{aligned} \mathrm{D}^+V = &-\frac{1}{r}(k_v\sum_{i=1}^N(\tanh\Delta_{i+} - \tanh\Delta_i) - \boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\Delta})L(\boldsymbol{\Delta})\tilde{\boldsymbol{\sigma}} \\ &+ \tilde{\boldsymbol{\sigma}}^{\mathrm{T}}L(\boldsymbol{\Delta})\boldsymbol{T}(\boldsymbol{\Delta})) + \tilde{\boldsymbol{u}}^{\mathrm{T}}(PS + S^{\mathrm{T}}P)\tilde{\boldsymbol{u}} \\ \le &-\frac{k_v}{r}\sum_{i=1}^N(\tanh\Delta_{i+} - \tanh\Delta_i) \le 0, \end{aligned} \tag{46}$$

where it is noted that $PS + S^{\mathrm{T}}P \le 0$ and $L(\boldsymbol{\Delta})$ is always positive semi-definite. Define set $\chi = \{[\boldsymbol{\Delta}^{\mathrm{T}}\ \tilde{\boldsymbol{u}}^{\mathrm{T}}]^{\mathrm{T}} | \dot{V}(\boldsymbol{\Delta}, \tilde{\boldsymbol{u}}) = 0\}$. Since $\dot{V}(\boldsymbol{\Delta}, \tilde{\boldsymbol{u}}) = 0$ implies $\Delta_1 = \Delta_{1+} = ... = \Delta_{1-}$, and $\Delta_1 + \Delta_{1+} + ... + \Delta_{1-} \equiv 0$ holds by (34), we have $\chi = \{[\boldsymbol{\Delta}^{\mathrm{T}}\ \tilde{\boldsymbol{u}}^{\mathrm{T}}]^{\mathrm{T}} | \boldsymbol{\Delta} = \boldsymbol{0}\}$. As $\boldsymbol{\Delta} = \boldsymbol{0}$ holds, $\dot{\boldsymbol{\Delta}} = 0$ yields $\tilde{\boldsymbol{\sigma}} = \tilde{\varpi}\boldsymbol{1}$ with some variable $\tilde{\varpi}$. Thus, then $\chi = \{[\boldsymbol{\Delta}^{\mathrm{T}}\ \tilde{\boldsymbol{u}}^{\mathrm{T}}]^{\mathrm{T}} | \boldsymbol{\Delta} = \boldsymbol{0},\ B^{\mathrm{T}}\tilde{\boldsymbol{u}} = \tilde{\varpi}\boldsymbol{1}\}$, and by the non-smooth LaSalle Invariance Principle in [47, Theorem 3.2, Chapter VII], $\boldsymbol{\Delta}(t)$ converges to $\boldsymbol{0}$, i.e., $\varphi_{(i+)i} - \varphi_{i(i-)}$, $\forall i \in \mathcal{O}$, converges to 0 asymptotically as $t \to \infty$. It follows from the definition of $\varphi_{(i+)i}$ that $\sum_{i=1}^N \varphi_{(i+)i} \equiv 2\pi$, and thus

$$\lim_{t\to\infty} \varphi_{(i+)i}(t) = \frac{2\pi}{N},\ \forall i \in \mathcal{O}. \tag{47}$$

Hence, Γ_2 is globally asymptotically stable relative to Γ_1. The proof is thus completed. ■

For the closed-loop system consisting of N nonlinear systems (11), (13), and control laws (17)–(21), only the asymptotical stability rather than the exponential one can be derived, and the convergence rate of the system states cannot be determined by k_v, k_ω, and K_i. This fact can be observed from (29), where it is shown that the decreasing rate of the Lyapunov function V_i is not influenced by the concerned states e_{yi} and $\tilde{\boldsymbol{z}}_i$. Nevertheless, it follows from (29) and (46) that larger values of k_v, k_ω, and the eigenvalues of $K_i + K_i^{\mathrm{T}}$ lead to faster decrease of the corresponding Lyapunov functions, which may result in a faster convergence rate.

Remark 3.1: In [39], one single mobile robot with input disturbances was considered, and the proposed control law for encircling the target, utilized both control channels v_i and ω_i. While for multiple mobile robots, an additional objective for evenly-spaced formation has to be achieved with the same control channels, which makes the proof of Theorem 1 more challenging than that in [39]. Similar to [39], the internal states $\hat{\rho}_i$ and $\hat{\varrho}_i$ are introduced to handle the input disturbances, in the control law for encircling the target. However, it can be observed from (13) and (17) that introducing $\hat{\rho}_i$ leads to another signal $\hat{\rho}_i - \rho_i$. As $\hat{\rho}_i - \rho_i$ is also unknown and may not vanish, we design another internal state $\hat{\sigma}_i$ as well as its update law (21) to handle it. Once v_i is enhanced with $\hat{\sigma}_i$, the stability analysis for encircling the target, i.e., Proposition 1, becomes more complicated. In particular, in order to prove (32), more effort has been paid to guarantee that $\bar{v}_i$ in (31) does not converge to zero. If $\hat{\sigma}_i$ were not introduced as in [39], $\bar{v}_i$ would equal v_0, which directly leads to (32) but makes it impossible to achieve evenly spaced formation. Besides, even with $\hat{\sigma}_i$ and its update law (21), system (13) will not evolve into (40) and Proposition 2 as well as Theorem 1 will no longer hold if (32) cannot be proved in Proposition 1. ■

C. Discussions

1) Disturbance signals: As explained in Remark 2.1, the disturbance described by the outputs of the exo-systems (2) under Assumption 1, are the linear combinations of a finite number of sinusoidal signals. Assumption 1 is a more relaxed assumption than that used in [35–38]. In [35, 36], a class of input disturbances described by the outputs of the exo-systems (2) satisfying the following assumption was considered.

Assumption 4: [35, 36] The matrices $\boldsymbol{b}_i\boldsymbol{b}_i^{\mathrm{T}}S_i + S_i^{\mathrm{T}}\boldsymbol{b}_i\boldsymbol{b}_i^{\mathrm{T}}$ and $\boldsymbol{c}_i\boldsymbol{c}_i^{\mathrm{T}}S_i + S_i^{\mathrm{T}}\boldsymbol{c}_i\boldsymbol{c}_i^{\mathrm{T}}$, $i \in \mathcal{O}$, are negative semi-definite. ■

As given in Remark 1 of [36], the input disturbance signal $[\rho_i\ \varrho_i]^{\mathrm{T}}$ will not diverge under Assumption 4. In [37, 38], a class of input disturbances consisting of harmonic and constant signals was considered, which implies that the exo-systems (2) satisfy the following assumption:

Assumption 5: [37, 38] The matrices S_i, $i \in \mathcal{O}$, are skew-symmetric, i.e., $S_i + S_i^{\mathrm{T}} = 0$. ■

Since many signals cannot be represented by linear combinations of a finite number of sinusoidal signals, the scope of the disturbance signals is limited by Assumptions 4 and 5. See the following example.

Example 1: Consider a disturbance signal $[\rho_i(t)\ \varrho_i(t)]^{\mathrm{T}} = [\frac{\sqrt{2}}{6}+\frac{1}{3}\sin(\frac{3}{5}t+\frac{\pi}{4})\quad \frac{\sqrt{2}}{6}+\frac{1}{15}\sin(\frac{3}{5}t+\frac{\pi}{4})-\frac{1}{5}\cos(\frac{3}{5}t+\frac{\pi}{4})]^{\mathrm{T}}$ which can be represented as the output of the exo-system (2) with $S_i = \begin{bmatrix} 0.2 & -1 & 0 \\ 0.4 & -0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{b}_i = [1\ 0\ 1]^{\mathrm{T}}$, $\boldsymbol{c}_i = [0\ 1\ 1]^{\mathrm{T}}$, and $\boldsymbol{z}_i(0) = [\frac{\sqrt{2}}{6}\ \ -\frac{\sqrt{2}}{15}\ \ \frac{\sqrt{2}}{6}]^{\mathrm{T}}$, which dissatisfies Assumptions 4 and 5, but satisfies Assumptions 1. ■

As in [35–39], S_i, $\boldsymbol{b}_i$, and $\boldsymbol{c}_i$ in the exo-system (2) are assumed to be known. For the concerned input disturbances (8)–(9), S_i, $\boldsymbol{b}_i$, and $\boldsymbol{c}_i$ can be determined by the frequencies γ_{ij} and γ'_{ij}. This theoretical framework may be suitable for the mobile robots subject to the input disturbances which are caused by the systematic errors. As for the systematic errors, these frequencies can be measured or estimated without knowing their exact values of magnitudes in some practical scenarios. The case that the system matrix S_i is unknown will lead to a much more complicated problem which needs further investigation and is our future research focus.

For the case where the dynamics of the disturbances are unknown and upper bounds on the magnitudes of the disturbances are known, sliding mode control technique can be used. However, this strategy usually leads to chattering. For instance, in [40] where consensus of system (1) was studied, the proposed controller leads to chattering of heading angles. Our proposed control law provide an option for the case where the frequency information rather than the upper bounds on the magnitudes of the disturbances is available, and where the chattering is not physically allowed.

2) Persistently exciting condition: Under Assumption 1, if the pair $(\boldsymbol{b}_i^{\mathrm{T}}, S_i)$ is observable and the unknown initial state satisfies $S_i\boldsymbol{z}_i(t_0) \neq \mathbf{0}$, then $\lim_{t\to\infty}\rho_i(t)$ does not exist. Since ρ_i is in the form of (8), there exists at least one $[\kappa_{ij}\quad \gamma_{ij}]^{\mathrm{T}} \neq \mathbf{0}$. As $\kappa_{ij}\sin(\gamma_{ij}t+\psi_{ij})$ satisfies the classic persistently exciting condition [49], the disturbance $\rho_i(t)$ is a persistent exciting. The condition that each pair $(\boldsymbol{b}_i^{\mathrm{T}}, S_i),\ i \in \mathcal{O}$, is observable, is a sufficient condition to solve Problem 1, and it is only used to ensure that $\hat{\sigma}_i(t)$ does not converge to a constant. If $\hat{\sigma}_i(t)$ converges to a constant $\hat{\varpi}_i$, then the value of $\hat{\varpi}_i$ cannot be determined and may result in $\bar{v}_i(t) \to 0$.

However, in the case where $\lim_{t\to\infty}\hat{\sigma}_i(t)$ does not converge to v_0, the proposed control laws are also effective even when the pair $(\boldsymbol{b}_i^{\mathrm{T}}, S_i),\ i \in \mathcal{O}$, is not observable. Thus, as the constant $\hat{\varpi}$ does not equal v_0 in most cases, the condition that $(\boldsymbol{b}_i^{\mathrm{T}}, S_i),\ i \in \mathcal{O}$, is observable, is conservative. In practice, if $\lim_{t\to\infty}\hat{\sigma}_i(t) = v_0$ happens, the robots may simply adjust the value of v_0 slightly to avoid $\hat{\sigma}_i(t) \to v_0$. In the case that $\boldsymbol{b}_i^{\mathrm{T}}S_i = \mathbf{0}^{\mathrm{T}}$, v_0 can be designed as a function of which the limit does not exist as $t \to \infty$. We may modify v_0, and obtain the following corollary.

Corollary 1: In the case that $\boldsymbol{b}_i^{\mathrm{T}}S_i = \mathbf{0}^{\mathrm{T}}$, the *cooperative target enclosing control problem*, i.e., Problem 1 can be solved by control law (17)–(21) under Assumptions 1–3 with v_0 designed as $v_0 = a_0 + b_0(t)$ where $\lim_{t\to\infty} b_0(t)$ does not exist and a_0 is a positive constant satisfying $a_0 >> \max b_0(t)$. ■

Sketch of proof: The proof can be proceeded as that of Theorem 1 as well as Propositions 1 and 2. The only difference is that $\hat{\sigma}_i$ converge to a constant $\hat{\varpi}$ by (39) and $\boldsymbol{b}_i^{\mathrm{T}}S_i = \mathbf{0}^{\mathrm{T}}$. Then, as $v_0 = \hat{\varpi}$, the design of v_0 in Corollary 1 guarantees that v_0 is not a constant, which leads to the contradiction.

3) Neighbouring robots: To achieve the evenly spaced formation of the robots, the network topology was often assumed to be a cycle, for instance, [14, 16, 19, 21, 26, 50, 51]. That is, each robot has two neighbors, robots $i+$ and $i-$. Note that those works does not include any disturbances, while the input disturbances are considered in this paper and Assumption 3 is needed when handling these disturbances. If there were no input disturbances, dynamics (42) would reduce to

$$\mathrm{D}^+\Delta_i = -\frac{1}{r}(k_v(2\tanh\Delta_i - \tanh\Delta_{i^+} - \tanh\Delta_{i^-}). \quad (48)$$

Then, only using controller (17) without $\hat{\rho}_i$ and $\hat{\sigma}_i$, i.e.,

$$v_i = v_0 + k_v\tanh(\varphi_{(i+)i} - \varphi_{i(i-)}), \quad (49)$$

can make N systems in the form of (48) globally asymptotically stable. In this case, each robot only needs two neighbors, robots $i+$ and $i-$, to implement (49), which consists with the assumption used in the aforementioned existing works. However, once the input disturbances are considered, there appears the disturbance $(2\tilde{\sigma}_i - \tilde{\sigma}_{i^+} - \tilde{\sigma}_{i^-})$ in (48), that is, (48) becomes (42). To handle this disturbance, we develop (21) which has to use the measurements $\varphi_{(i++)i}$ and $\varphi_{i(i--)}$. Thus, in addition to robots $i+$ and $i-$, robot i needs to have another two neighbors, robots $i{+}{+}$ and $i{-}{-}$.

4) Collision: To avoid collision among mobile robots in the same altitude, we can use a switched law similar to that in [16] by incorporating the proposed control law with the behavior-based algorithm implemented on the experiment in [16]. That is, when two robots are going to collide and satisfy a certain condition in the algorithm, they switch to the controller for avoiding collision used in [16]. Then, they switch back to our proposed controller once they do not tend to collide. Note that when $[\boldsymbol{e}^{\mathrm{T}}\ \hat{\boldsymbol{e}}^{\mathrm{T}}]^{\mathrm{T}}$ is in a small neighborhood of Γ_2, i.e., every $\varphi_{(i+)i}$ remains close to $\frac{2\pi}{N}$, the collision will not happen, since collision only occurs when $\varphi_{(i+)i}$ turns zero.

IV. Simulation

In this section, we consider five mobile robots (1) with the initial states $\boldsymbol{p}_1(0) = [-3\ 3]^{\mathrm{T}}$, $\boldsymbol{p}_2(0) = [-4\ 2]^{\mathrm{T}}$, $\boldsymbol{p}_3(0) = [-4\ -4]^{\mathrm{T}}$, $\boldsymbol{p}_4(0) = [4\ -4]^{\mathrm{T}}$, $\boldsymbol{p}_5(0) = [3\ 3]^{\mathrm{T}}$, $\boldsymbol{p}_6(0) = [0\ 4]^{\mathrm{T}}$, $\theta_1(0) = \frac{\pi}{4}$, $\theta_2(0) = -\frac{\pi}{2}$, $\theta_3(0) = \frac{2\pi}{3}$, $\theta_4(0) = \frac{\pi}{3}$, $\theta_5(0) = -\frac{2\pi}{3}$, and $\theta_6(0) = \frac{\pi}{2}$. The position of the target and the radius are given by $\boldsymbol{p}_0 = [0\ \ 0]^{\mathrm{T}}$ and $r = 2$ respectively. Give $v_0 = 2$, and set $K_i = \mathrm{diag}(20, 20)$ and the parameters $k_v = 3$ and $k_\omega = 10$.

In this example, nonidentical input disturbances of the multi-robot systems are considered. The disturbance signals are $[\rho_i(t)\ \ \varrho_i(t)]^{\mathrm{T}} = [\frac{\sqrt{2}}{6} + \frac{1}{3}\sin(\frac{3(6-i)}{10}t + \frac{\pi}{4})\qquad \frac{\sqrt{2}}{6} + \frac{1}{15}\sin(\frac{3(6-i)}{10}t + \frac{\pi}{4}) - \frac{1}{5}\cos(\frac{3(6-i)}{10}t + \frac{\pi}{4})]^{\mathrm{T}},\ i = 1, 2, ..., 6$, which describe a set of persistent and periodical disturbances caused by some systematic errors of the cruise control systems in the mobile robots. The concerned disturbance signals are in the form of (8)–(9) and can be viewed as the outputs of the

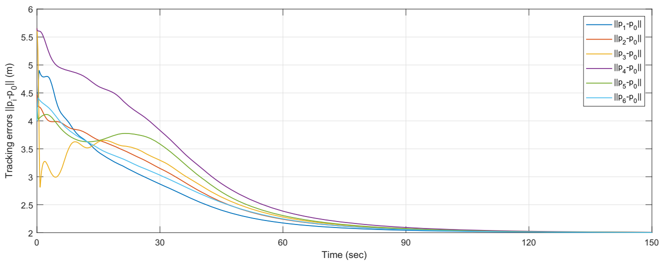


Fig. 2. The relative distances $\|\boldsymbol{p}_i - \boldsymbol{p}_0\|$, $i = 1, .., 6$, during 0–150s.

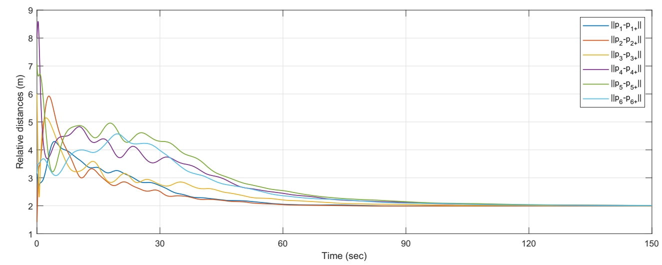


Fig. 3. The relative distances $\|\boldsymbol{p}_i(t) - \boldsymbol{p}_{i+}(t)\|$, $i = 1, .., 6$, during 0–150s.

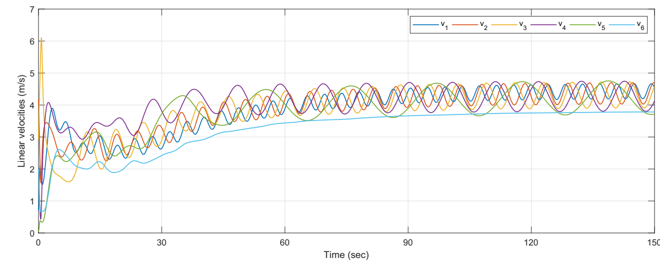


Fig. 4. The linear velocities v_i, $i = 1, .., 6$, during 0–150s.

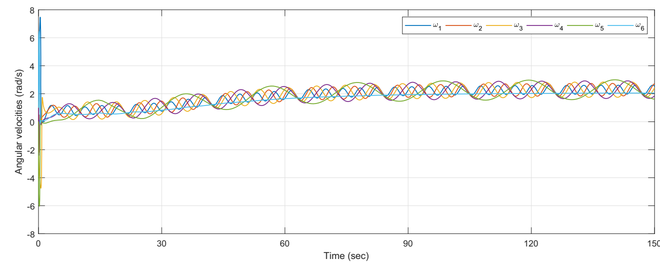


Fig. 5. The angular velocities ω_i, $i = 1, .., 6$, during 0–150s.

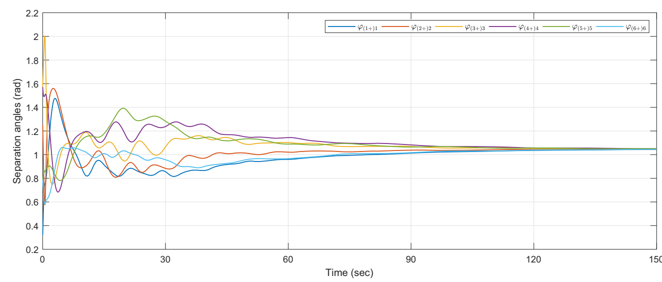


Fig. 6. The separation angles $\varphi_{(i+)i}$, $i = 1, .., 6$, during 0–150s.

exogenous systems (2) with $S_i = (6-i)\times \begin{bmatrix} 0.1 & -0.5 & 0 \\ 0.2 & -0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{b}_i = [1\ 0\ 1]^{\mathrm{T}}$, $\boldsymbol{c}_i = [0\ 1\ 1]^{\mathrm{T}}$, and initial states $\boldsymbol{z}_i(0) = [\frac{\sqrt{i}}{6}\quad -\frac{\sqrt{i}}{15}\quad \frac{\sqrt{i}}{6}]^{\mathrm{T}}$. It is verified that Assumption 1 is satisfied. Besides, each pair $(\boldsymbol{b}_i^{\mathrm{T}}, S_i)$, $i = 1, .., 5$, is observable. By (22), we set $P_i = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $i = 1, 2, ..., 6$.

Apply the proposed control law (17)–(21) to each robot (1) with the initial internal states $\hat{\boldsymbol{e}}_i(0) = \mathbf{0}$, $\hat{\boldsymbol{z}}_i(0) = \mathbf{0}$ and $\hat{\boldsymbol{u}}_i(0) = \mathbf{0}$, $i = 1, .., 6$. The relative distances between each robot i and the target during 0–150s are presented in Fig. 2, which shows that robots converge to the common circle with the given center $\boldsymbol{p}_0$ and radius r. The distances between each robot i and its neighbor $i+$ during 0–150s are shown in Fig. 3, which illustrates that the mobile robots converge to the evenly spaced formations. The positions of all robots at $t = \{0, 5, 10, 15, 30, 60, 90, 120, 150\}s$ are shown in Fig. 7, which shows the cooperative target enclosing are eventually achieved. The control inputs, i.e., the linear velocity and angular velocity of each robot i are shown in Figs. 4 and 5, respectively. The control inputs change according to the proposed controller (17)–(18) and eventually vary properly so as to achieve the rejection to the input disturbances. Besides, the separation angles $\varphi_{(i+)i}$ during 0–150s are presented in Fig. 6. Note that the neighbours of the robots may dynamic switch depending on the real-time counterclockwise radial order around the target. As shown in Fig. 6, no separation angles $\varphi_{(i+)i}$, $i = 1, .., 6$, have ever turned zero, which indicates that no switch of neighbors happens in this example. Furthermore, it can be observed from Figs. 3 and 6 that no collision among the robots has happened in this example, since neither $\|\boldsymbol{p}_i(t) - \boldsymbol{p}_{i+}(t)\|$ nor $\varphi_{(i+)i}$, $i = 1, .., 6$, has ever reached zero.

As in [35, 36], we include the case $S_6 = 0$ which satisfies both Assumptions 1 and 4. In this case, the disturbances are constant. It is shown that even if the pair $(\boldsymbol{b}_6^{\mathrm{T}}, S_6)$ is not observable, the cooperative target enclosing are still achieved. As discussed above, this condition is a conservative assumption only used to exclude $\lim_{t\to\infty} \hat{\sigma}_i(t) = v_0$, so as to guarantee the solvability of Problem 1. In most cases, control law (17)–(21) are still effective as generally $\hat{\sigma}_i(t)$ does not converge to v_0.

V. Conclusions

In this paper, we have proposed a dynamic control law, such that a group of unicycle-type mobile robots enclose and orbit around a given target while maintaining evenly spaced along the circle, in the presence of heterogeneous input disturbances. The network topology is set in a cyclic pursuit manner. The proposed control law is based on relative displacement measurements, and guarantees the global asymptotical stability of the closed-loop multi-robot systems under certain assumptions. For the future work, we will consider the disturbances generated from an exo-system with unknown system matrix, investigate the rejection to the non-periodical disturbances caused by some nondeterministic errors of the robots, and study the moving-target enclosing control problem of mobile robots with input disturbances.

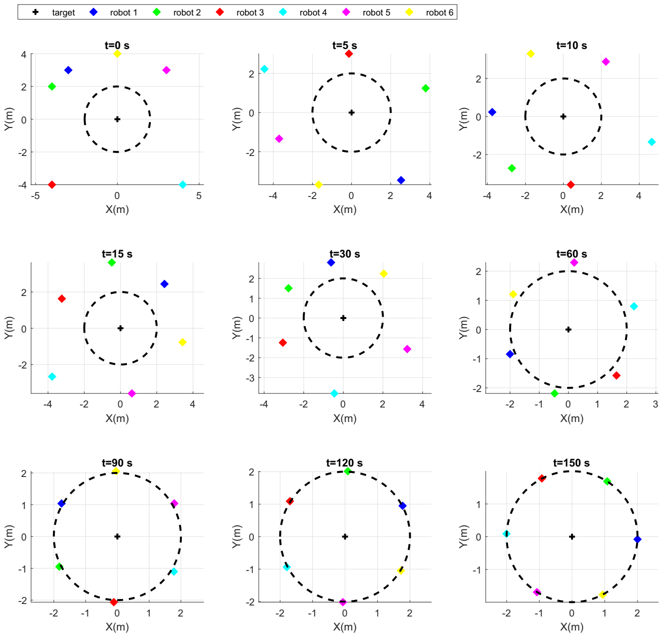


Fig. 7. The positions of the target and all mobile robots at $t = \{0, 5, 10, 15, 30, 60, 90, 120, 150\}s$.

REFERENCES

[1] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[2] L. Liu, D. Wang, Z. Peng, and T. Li, "Modular adaptive control for los-based cooperative path maneuvering of multiple underactuated autonomous surface vehicles," *IEEE Trans. Syst. Man Cybern., Syst.*, vol. 47, no. 7, pp. 1613–1624, 2017.

[3] Z. Miao, Y.-H. Liu, Y. Wang, G. Yi, and R. Fierro, "Distributed estimation and control for leader-following formations of nonholonomic mobile robots," *IEEE Trans. Autom. Sci. Eng.*, no. 99, pp. 1–9, 2018.

[4] H. Liu, Y. Wang, and F. L. Lewis, "Robust distributed formation controller design for a group of unmanned underwater vehicles," *IEEE Trans. Syst. Man Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2019.2895499.

[5] C.-W. Kuo, C.-C. Tsai, and C.-T. Lee, "Intelligent leader-following consensus formation control using recurrent neural networks for small-size unmanned helicopters," *IEEE Trans. Syst. Man Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2019.2896958.

[6] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis, "Collective motion, sensor networks, and ocean sampling," *Proc. IEEE*, vol. 95, no. 1, pp. 48–74, 2007.

[7] Z.-P. Jiang and H. Nijmeijer, "Tracking control of mobile robots: a case study in backstepping," *Automatica*, vol. 33, no. 7, pp. 1393–1399, 1997.

[8] D. Chwa, "Tracking control of differential-drive wheeled mobile robots using a backstepping-like feedback linearization," *IEEE Trans. Syst. Man Cybern. Part A Syst. Humans*, vol. 40, no. 6, pp. 1285–1295, 2010.

[9] J. Fu, F. Tian, T. Chai, Y. Jing, Z. Li, and C.-Y. Su, "Motion tracking control design for a class of nonholonomic mobile robot systems," *IEEE Trans. Syst. Man Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2018.2804948.

[10] Z. Qu, *Cooperative control of dynamical systems: applications to autonomous vehicles*. Springer, 2009.

[11] R. Sepulchre, D. A. Paley, and N. E. Leonard, "Stabilization of planar collective motion: All-to-all communication," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 811–824, 2007.

[12] ——, "Stabilization of planar collective motion with limited communication," *IEEE Trans. Autom. Control*, vol. 53, no. 3, pp. 706–719, 2008.

[13] N. Ceccarelli, M. Di Marco, A. Garulli, and A. Giannitrapani, "Collective circular motion of multi-vehicle systems," *Automatica*, vol. 44, no. 12, pp. 3025–3035, 2008.

[14] Y. Lan, G. Yan, and Z. Lin, "Distributed control of cooperative target enclosing based on reachability and invariance analysis," *Syst. Control Lett.*, vol. 59, no. 7, pp. 381–389, 2010.

[15] N. Moshtagh, N. Michael, A. Jadbabaie, and K. Daniilidis, "Vision-based, distributed control laws for motion coordination of nonholonomic robots," *IEEE Trans. Rob.*, vol. 25, no. 4, pp. 851–860, 2009.

[16] R. Zheng, Y. Liu, and D. Sun, "Enclosing a target by nonholonomic mobile robots with bearing-only measurements," *Automatica*, vol. 53, pp. 400–407, 2015.

[17] Z. Chen and H.-T. Zhang, "No-beacon collective circular motion of jointly connected multi-agents," *Automatica*, vol. 47, no. 9, pp. 1929–1937, 2011.

[18] ——, "A remark on collective circular motion of heterogeneous multi-agents," *Automatica*, vol. 49, no. 5, pp. 1236–1241, 2013.

[19] J. A. Marshall, M. E. Broucke, and B. A. Francis, "Formations of vehicles in cyclic pursuit," *IEEE Trans. Autom. Control*, vol. 49, no. 11, pp. 1963–1974, 2004.

[20] ——, "Pursuit formations of unicycles," *Automatica*, vol. 42, no. 1, pp. 3–12, 2006.

[21] X. Yu and L. Liu, "Distributed circular formation control of ring-networked nonholonomic vehicles," *Automatica*, vol. 68, pp. 92–99, 2016.

[22] M. I. El-Hawwary and M. Maggiore, "Distributed circular formation stabilization for dynamic unicycles," *IEEE Trans. Autom. Control*, vol. 58, no. 1, pp. 149–162, 2013.

[23] X. Yu, X. Xu, L. Liu, and G. Feng, "Circular formation of networked dynamic unicycles by a distributed dynamic control law," *Automatica*, vol. 89, pp. 1–7, 2018.

[24] A. Sinha and D. Ghose, "Generalization of nonlinear cyclic pursuit," *Automatica*, vol. 43, no. 11, pp. 1954–1960, 2007.

[25] G. S. Seyboth, J. Wu, J. Qin, C. Yu, and F. Allgower, "Collective circular motion of unicycle type vehicles with non-identical constant velocities," *IEEE Trans. Control Network Syst.*, vol. 1, no. 2, pp. 167–176, 2014.

[26] R. Zheng, Z. Lin, M. Fu, and D. Sun, "Distributed control for uniform circumnavigation of ring-coupled unicycles," *Automatica*, vol. 53, pp. 23–29, 2015.

[27] W. Dixon, D. Dawson, and E. Zergeroglu, "Tracking and regulation control of a mobile robot system with kinematic disturbances: A variable structure-like approach," *J. Dyn. Sys., Meas., Control*, vol. 122, pp. 616–623, 2000.

[28] D. Wang and C. B. Low, "Modeling and analysis of skidding and slipping in wheeled mobile robots: Control design perspective," *IEEE Trans. Rob.*, vol. 24, no. 3, pp. 676–687, 2008.

[29] A. Liu, W.-A. Zhang, L. Yu, H. Yan, and R. Zhang, "Formation control of multiple mobile robots incorporating an extended state observer and distributed model predictive approach," *IEEE Trans. Syst. Man Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2018.2855444.

[30] W. Deng and J. Yao, "Adaptive integral robust control and application to electromechanical servo systems," *ISA Trans.*, vol. 67, pp. 256–265, 2017.

[31] T. Dierks and S. Jagannathan, "Neural network control of mobile robot formations using rise feedback," *IEEE Trans. Syst. Man Cybern. Part B Cybern.*, vol. 39, no. 2, pp. 332–347, 2009.

[32] B. Xiao, X. Yang, H. R. Karimi, and J. Qiu, "Asymptotic tracking control for a more representative class of uncertain nonlinear systems with mismatched uncertainties," *IEEE Trans. Ind. Electron.*, to be published, doi: 10.1109/TIE.2019.2893852.

[33] L. Cao, D. Qiao, and X. Chen, "Laplace ℓ_1 huber based cubature kalman filter for attitude estimation of small satellite," *Acta Astronaut.*, vol. 148, pp. 48–56, 2018.

[34] L. Cao, D. Ran, X. Chen, X. Li, and B. Xiao, "Huber second-order variable structure predictive filter for satellites attitude estimation," *Int. J. Control Autom. Syst.*, to be published, doi: 10.1007/s12555-018-0804-4.

[35] A. Ajorlou, M. M. Asadi, A. G. Aghdam, and S. Blouin, "A consensus control strategy for unicycles in the presence of disturbances," in *Proc. 2013 Am. Control Conf.*, Washington, DC, USA, 2013, pp. 4039–4043.

[36] C. Yang, F. Chen, L. Xiang, and W. Lan, "Distributed rendezvous and tracking for multiple unicycles with heterogeneous input disturbances," *Int. J. Robust Nonlinear Control*, vol. 27, no. 9, pp. 1589–1606, 2017.

[37] M. Jafarian, E. Vos, C. De Persis, J. Scherpen, and A. Schaft, "Disturbance rejection in formation keeping control of nonholonomic wheeled robots," *Int. J. Robust Nonlinear Control*, vol. 26, no. 15, pp. 3344–3362, 2016.

[38] M. Jafarian, "Robust consensus of unicycles using ternary and hybrid controllers," *Int. J. Robust Nonlinear Control*, vol. 27, no. 17, pp. 4013–4034, 2017.

[39] X. Yu and L. Liu, "Target enclosing and trajectory tracking for a mobile robot with input disturbances," *IEEE Control Syst. Lett.*, vol. 1, no. 2, pp. 221–226, 2017.

[40] A. Ajorlou, M. M. Asadi, A. G. Aghdam, and S. Blouin, "Distributed

consensus control of unicycle agents in the presence of external disturbances," *Syst. Control Lett.*, vol. 82, pp. 86–90, 2015.

[41] Y. Cheng, R. Jia, H. Du, G. Wen, and W. Zhu, "Robust finite-time consensus formation control for multiple nonholonomic wheeled mobile robots via output feedback," *Int. J. Robust Nonlinear Control*, vol. 28, pp. 2082–2096, 2018.

[42] Z. Lin, M. Broucke, and B. Francis, "Local control strategies for groups of mobile autonomous agents," *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 622–629, 2004.

[43] T.-H. Kim and T. Sugie, "Cooperative control for target-capturing task based on a cyclic pursuit strategy," *Automatica*, vol. 43, no. 8, pp. 1426–1431, 2007.

[44] Y. Su and J. Huang, "Cooperative output regulation with application to multi-agent consensus under switching network," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 42, no. 3, pp. 864–875, 2012.

[45] M. I. El-Hawwary and M. Maggiore, "Reduction theorems for stability of closed sets with application to backstepping control design," *Automatica*, vol. 49, no. 1, pp. 214–222, 2013.

[46] W. E. Dixon, D. M. Dawson, E. Zergeroglu, and A. Behal, *Nonlinear control of wheeled mobile robots*. Springer-Verlag New York, Inc., 2001.

[47] N. Rouche, P. Habets, M. Laloy, and A. M. Ljapunov, *Stability theory by Liapunov's direct method*. Springer-Verlag New York, Inc., 1977.

[48] I. Barkana, "Defending the beauty of the invariance principle," *Int. J. Control*, vol. 87, no. 1, pp. 186–206, 2014.

[49] K. S. Narendra and A. M. Annaswamy, "Persistent excitation in adaptive systems," *Int. J. Control*, vol. 45, no. 1, pp. 127–160, 1987.

[50] Z. Lin, B. A. Francis, and M. Maggiore, "Necessary and sufficient graphical conditions for formation control of unicycles," *IEEE Trans. Autom. Control*, vol. 50, no. 1, pp. 121–127, 2005.

[51] T.-H. Kim, S. Hara, and Y. Hori, "Cooperative control of multi-agent dynamical systems in target-enclosing operations using cyclic pursuit strategy," *Int. J. Control*, vol. 83, no. 10, pp. 2040–2052, 2010.

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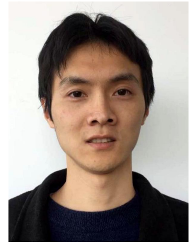

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