Bearing-Only Formation Tracking Control of Multi-Agent Systems With Local Reference Frames and Constant-Velocity Leaders

Jianing Zhao, Xiao Yu, Xianwei Li, and Hesheng Wang

***Abstract*—In this paper, the bearing-only formation tracking control problem of multi-agent systems modeled as double-integrators in local reference frames is investigated. The sensing topology among all agents is described by a multiply rooted undirected graph and the autonomous leaders associated with a common reference frame move with a constant velocity. The proposed control law for each follower depends merely on the relative bearings and relative orientations to its neighbors in its local reference frame. The orientation synchronization problem is first solved so that all orientations of the followers converge to that of the leaders. Then, a bearing-only controller is developed to achieve the desired moving formation. Finally, an example illustrates the effectiveness.**

***Index Terms*—Cooperative control, control of networks, constrained control.**

I. INTRODUCTION

FORMATION control of multi-agent systems has attracted much research attention recently, and the existing approaches are categorized into position-based, displacement-based, distance-based and bearing-based schemes [1]. In particular, the bearing-based one provides much practicability, since bearing measurements can be obtained by a passive sensor, such as the onboard optical cameras [2].

Bearing-only formation control is merely based on the bearing measurements of the neighbors without any auxiliary distance-or-other measurements or communication, which yields many challenges [3]–[5]. In [6], the bearing rigidity theory was put forward to solve the uniqueness problem of a geometrical pattern with bearing constraints in an arbitrary dimension. Besides, the angle rigidity theory was raised in [7] to investigate the uniqueness of a formation with angle constraints on a two-dimensional plane. Based on the notion of bearing rigidity, many important problems such as bearing-only formation stabilization [6], [8]–[10], bearing-based formation maneuver [11]–[13] and bearing-based network localization [14], [15] have been studied. Those bearing-only or bearing-based formation control approaches, however, are either only applicable to the formation with stationary targets, or in need of the relative positions and velocities. Moreover, it is of practical importance to investigate the case with moving targets. The bearing-only tracking control problem of single-integrators, double-integrators and unicycles was solved in [16], where the control laws are designed in global reference frames. Nonetheless, the global reference frames may not be accessible to agents in some scenarios such as the indoor cases.

To solve the formation control problem in the absence of global reference frames, the orientation estimation or synchronization is needed. In [17], a finite-time distributed global orientation estimation law was proposed with an auxiliary matrix transmitted between neighboring agents via communication. In [18], a passivity-based distributed angular velocity controller using neighbors' relative orientations was developed to achieve orientation synchronization in $SE(3)$. This result was adopted in [6] to achieve a stationary target formation. However, the proof whether this controller is applicable to the leader-following case is not given in [18].

This paper aims to solve the leader-following formation tracking problem of double-integrators, and the bearing measurements are measured in each follower agent's body-fixed local reference frame. The sensing topology among all agents is described by a multiply rooted undirected graph in which the roots denote the leaders. First, the orientation synchronization is achieved so that the multi-agent systems are able to share a common orientation. Then, a bearing-only formation tracking control law is proposed to achieve a desired moving formation by using the bearings measured in local reference frames.

The main contribution is summarized as follows. Firstly, the proposed control law only requires each follower agent to use its own local reference frame, in contrast to [16] where all agents need to share a common reference frame, i.e., the global reference frame. Secondly, a rigorous proof of the asymptotic stability with locally exponential convergence of the orientation synchronization with multiply leaders is presented, while the convergence rate in the leader-following case was not provided in [18].

The rest of this paper is organized as follows. The problem formulation and some necessary preliminaries are presented in Section II. The proposed control laws and stability analysis are shown in Section III. The simulation results are given in Section IV, and the conclusion is drawn in Section V.

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II. PRELIMINARIES AND PROBLEM FORMULATION

A. Notations and Preliminaries

Consider n mobile agents in $\mathbb{R}^d$, $d = 2$ or 3. Suppose that the first $n_l \geq 2$ agents are leaders and the rest $n_f = n - n_l$ agents are followers. Let $\Sigma_i,\ i \in \{1, \dots, n\}$ be the body-fixed local reference frame, whose origin is located at the center of agent i. Specifically, leaders are assumed to move with a common constant velocity and thus share a common reference frame, i.e., Σ_l. Denote the position, velocity in Σ_l and Σ_i of agent i as $p_i, v_i \in \mathbb{R}^d$ and $p_i^i, v_i^i \in \mathbb{R}^d$ respectively. Throughout this paper, the superscript i is used to express the vector quantities in Σ_i. Each follower agent is modeled as a double-integrator, i.e.,

$$\dot{p}_i^i = v_i^i, \quad \dot{v}_i^i = u_i^i, \tag{1}$$

where u_i^i is the acceleration control input to be designed based on the measurements in Σ_i.

Denote the configuration of the agents as $p = [p_1^\top, \dots, p_n^\top]^\top \in \mathbb{R}^{dn}$. The interaction among the agents is described by a multiply rooted undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a node set $\mathcal{V} = \{1, \dots, n\}$ and an edge set $\mathcal{E} = \{(j, i) : j \neq i, i, j \in \mathcal{V}\}$. The edge $(i, j) \in \mathcal{E}$ represents that the information of agent j can be measured by agent i. Denote the set of neighbors of agent i as $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. The number of agents in set $\mathcal{N}_i$ is denoted as $|\mathcal{N}_i|$. Let $\mathcal{V} = \mathcal{V}_l \cup \mathcal{V}_f$, where $\mathcal{V}_l = \{1, \dots, n_l\}$ and $\mathcal{V}_f = \{n_l + 1, \dots, n\}$ represent the sets of leaders and followers, respectively. Nodes in set $\mathcal{V}_l$ are viewed as roots and there is no edge between any two roots. Therefore, $\mathcal{G}$ is an n_l-rooted undirected graph. The edges between sets $\mathcal{V}_l$ and $\mathcal{V}_f$ are unidirectional, i.e., $\mathcal{V}_l \to \mathcal{V}_f$ while the edges among set $\mathcal{V}_f$ are bidirectional. Denote p_l, p_f, v_l and v_f as the positions and velocities of leaders and followers respectively. Denote the formation of agents as $\mathcal{G}(p)$ where node i of $\mathcal{G}$ is mapped to p_i for all $i \in \mathcal{V}$ and thus undirected graph $\mathcal{G}(p_f) \subset \mathcal{G}(p)$ is denoted as the formation of followers.

For every edge $(i, j) \in \mathcal{E}$, define the edge vector and bearing vector as $e_{ij} := p_j - p_i$ and $g_{ij} := \frac{e_{ij}}{\|e_{ij}\|}$ respectively, where $\|\cdot\|$ denotes the Euclidean norm of a vector or the spectral norm of a matrix. Define the orthogonal projection matrix $P_{g_{ij}} := I_d - g_{ij} g_{ij}^\top$, where $I_d \in \mathbb{R}^{d \times d}$ is the identity matrix. For convenience, express the vectors of the n_l-rooted undirected graph in a compact form. Let m be the number of edges in $\mathcal{G}$. Define the k-th edge in $\mathcal{G}$ as $e_k := e_{ij},\ g_k := \frac{e_k}{\|e_k\|}$, where $k \in \{1, \dots, m\}$. Let $e = [e_1^\top, \dots, e_m^\top]^\top$ and $g = [g_1^\top, \dots, g_m^\top]^\top$. Define $H \in \mathbb{R}^{m \times n}$ as the incidence matrix of $\mathcal{G}$ as in [6]. By definition, $e = (H \otimes I_d) p := \bar{H} p$, where $\otimes$ denotes the Kronecker product. Define the Laplacian matrix of the undirected graph $\mathcal{G}(p_f)$ describing the underlying topology of followers, as in [18], where all the edges have unit weights. $\mathcal{L}_f$ is positive semi-definite. Let $\lambda_{min2}(\mathcal{L}_f) > 0$ be the second smallest eigenvalue of $\mathcal{L}_f$.

There is a rotation between reference frames Σ_i and Σ_l. Without loss of generality, suppose all frames are three-dimensional (3D), i.e., $d = 3$, whereas the 2D case can be expressed by annihilating one dimensional coordinate in the 3D case. Let $SO(3) = \{Q \in \mathbb{R}^3 : Q^\top Q = I_3,\ det(Q) = 1\}$ be the special orthogonal group. Denote $Q_i \in SO(3)$ as the rotation matrix from Σ_i to Σ_l. Define the orientation of agent i as Q_i, i.e., the rotation relative to the attitude of leaders. Let $[\alpha_i, \beta_i, \gamma_i]^\top$ be the ZYX Euler angle of rotation Q_i and θ_i be the corresponding spatial angle between the attitude of agent i and that of leaders. The relation between $Q_i(\theta_i)$ and $[\alpha_i, \beta_i, \gamma_i]^\top$ is $Q_i(\theta_i) = R_z(\alpha_i) R_y(\beta_i) R_x(\gamma_i)$, where R_x, R_y and R_z represent the rotation matrix w.r.t. axes X, Y, and Z respectively. The relations between the velocities, bearings in Σ_l and Σ_i [6], [17], as well as the relation between the bearings' derivatives in Σ_l and Σ_i are

$$v_i = Q_i v_i^i, \quad g_{ij} = Q_i g_{ij}^i, \quad \dot{g}_{ij} = \dot{Q}_i g_{ij}^i + Q_i \dot{g}_{ij}^i, \tag{2}$$

where g_{ij}^i is the bearing measured in Σ_i.

The notation "$\wedge$" used in this paper is the skew-symmetric operator from a vector in $\mathbb{R}^3$ to the skew-symmetric matrix in $\mathbb{R}^{3\times 3}$, that is,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^\wedge = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{3}$$

The notation "$\vee$" denotes the inverse operator of "$\wedge$". Then, the dynamics of Q_i is

$$\dot{Q}_i = Q_i \hat{w}_i^i, \tag{4}$$

where w_i^i is the angular velocity to be designed in Σ_i. Define the energy of rotation $\phi(Q_i) := \frac{1}{2} tr(I_3 - Q_i) \geq 0$ as in [18], which represents "how much" Σ_i has rotated from Σ_l. Note that $\phi(Q_i) = 0$ if and only if $Q_i = I_3$, and the derivative of $\phi(Q_i)$ is $\dot{\phi}(Q_i) = (sk(Q_i)^\vee)^\top w_i^i$, with $sk(Q_i) := \frac{1}{2}(Q_i - Q_i^\top)$ according to [18].

Moreover, the concept of generalized positive definiteness [18] is used to describe a property of a real and not necessarily matrix. For a real matrix A, denote $A \succ 0$ if $x^\top A x > 0$ for all nonzero vectors x. Consequently, if $A \succ 0$, then the symmetric matrix $A + A^\top$ is positive definite.

B. Problem Formulation

The desired target formation of the multi-agent systems is formally defined as follows.

Definition 1: (*Target Formation*) Let $\mathcal{G}(p^*(t))$ be a target formation satisfying the following requirements:

1) Bearing: $\frac{p_j^*(t) - p_i^*(t)}{\|p_j^*(t) - p_i^*(t)\|} = g_{ij}^*,\ \forall (i, j) \in \mathcal{E}$.
2) Velocity: $\dot{p}^*(t) = v^* = \mathbf{1}_n \otimes v_c,\ v_c \in \mathbb{R}^d$,

where g_{ij}^* is called the bearing constraint. ■

Note that the autonomous leaders move with constant velocity v_c for $t \geq 0$. Substituting (1) and (4) to (2) yields the dynamics of the follower agent $i \in \mathcal{V}_f$ in Σ_l:

$$\dot{p}_i = v_i, \quad \dot{v}_i = \dot{Q}_i v_i + Q_i u_i^i, \quad \dot{Q}_i = Q_i \hat{w}_i^i. \tag{5}$$

The bearing-only formation tracking control problem of multi-agents systems in local reference frames is formulated as follows.

Problem 1: Consider n agents and an interaction graph $\mathcal{G}$. Given the constant bearing constraints g_{ij}^*, $(i, j) \in \mathcal{E}$, the leaders moving with a constant velocity v_c and the orientation

$Q_l(t) \equiv I_3,\ l \in \mathcal{V}_l$, for each follower $i \in \mathcal{V}_f$ with dynamics (5) and the initial position $p_i(t_0)$, velocity $v_i(t_0)$, orientation $Q_i(t_0)$, find a control law in the form of

$$u_i^i = \rho(g_{ij}^i, \dot{g}_{ij}^i, Q_i^{\mathrm{T}} Q_j),\ \ w_i^i = \varrho(Q_i^{\mathrm{T}} Q_j),\ \ j \in \mathcal{N}_i, \tag{6}$$

such that $p(t) \to p^*(t)$, $v(t) \to \mathbf{1}_n \otimes v_c$, $g(t) \to g^*$, and $Q_i(t) \to I_3$ as $t \to \infty$, where g_{ij}^i, $\dot{g}_{ij}^i$ and $Q_i^{\mathrm{T}} Q_j$ represent the local bearing, the varying rate of the local bearing and the relative orientation, respectively, and $\rho(\cdot)$ and $\varrho(\cdot)$ are sufficiently smooth functions to be designed. ■

The solution is based on the following assumptions.

[A1] $\mathcal{G}$ is fixed and connected.

[A2] $Q_i(t_0) \succ 0, i \in \mathcal{V}_f$.

[A3] $[p^{\mathrm{T}}(t_0), v^{\mathrm{T}}(t_0)]^{\mathrm{T}}$ locates in any a *priori* given (arbitrarily large) bounded set Ξ_0.

[A4] The bearing constraints g_{ij}^* and the leaders with a constant velocity v_c ensure a unique target formation $\mathcal{G}(p^*(t))$.

Remark 2.1: For the rotation matrix Q_i, $Q_i \succ 0$ if and only if the corresponding spatial angle θ_i satisfies $|\theta_i| < \frac{\pi}{2}$ [18]. Therefore, [A2] holds if the initial spatial rotation angle satisfies $|\theta_i(t_0)| < \frac{\pi}{2}$. ■

Remark 2.2: Assumptions similar to [A1] and [A2] were made in [18] to achieve the synchronization of the agents' orientations. [A3] is used to build the practical semi-global asymptotic stability. [A4] ensures a unique target formation. Otherwise, it is not guaranteed to achieve the target formation by any control approaches. To satisfy [A4], the bearing constraints and the leaders need to meet the condition in [14, Theorem 1]. ■

III. MAIN RESULTS

In this section, a bearing-only control law is proposed to solve the formation tracking control problem, i.e., Problem 1. A technical lemma in the Appendix, is developed as a corollary of [19, Lemma 2.1] to prove the semi-global asymptotic stability of the formation. Before this, the proof of locally exponential convergence of orientation synchronization is provided to ensure the condition in this lemma.

A. A Bearing-Only Control Law

The proposed acceleration and orientation control laws are

$$u_i^i = k_p \sum_{j\in\mathcal{N}_i} (g_{ij}^i - \frac{1}{2}(I_3 + Q_i^{\mathrm{T}} Q_j) g_{ij}^*) + k_v \sum_{j\in\mathcal{N}_i} \dot{g}_{ij}^i, \tag{7}$$

$$w_i^i = k_w \sum_{j\in\mathcal{N}_i} sk(Q_i^{\mathrm{T}} Q_j)^{\vee}, \quad i \in \mathcal{V}_f, \tag{8}$$

where k_p, k_v and k_w are any positive constant control gains.

The acceleration controller (7) is motivated by that in [16, Section IV]. The followers are supposed to be equipped with an optical pin-hole modeled camera. The varying rate of bearing $\dot{g}_{ij}^i$ in (7) can be obtained based on the pin-hole camera model, as the bearing g_{ij}^i is measured. Besides, the relative orientation $Q_i^{\mathrm{T}} Q_j$ can be obtained, for example, by a Kalman filtering estimator using line-of-sight measurement from an onboard optical sensor [20]. As the proposed dynamic control law relies merely on the local measurements and does not require any communications between agents, it can be implemented in a decentralized manner.

B. Stability Analysis of Formation

By substituting the control laws (7) and (8) into (5) and (2), the closed-loop system expressed in Σ_l is

$$\begin{aligned} \dot{p}_i &= v_i, \\ \dot{v}_i &= k_p \sum_{j\in\mathcal{N}_i} (g_{ij} - g_{ij}^*) + k_v \sum_{j\in\mathcal{N}_i} \dot{g}_{ij} + h_i(v_i, g_{ij}, \tilde{E})_{j\in\mathcal{N}_i}, \\ \dot{Q}_i &= \frac{k_w}{2} \sum_{j\in\mathcal{N}_i} Q_i(Q_i^{\mathrm{T}} Q_j - Q_j^{\mathrm{T}} Q_i), \end{aligned} \tag{9}$$

where $\tilde{E} = [E_1^{\mathrm{T}}, \dots, E_{n_f}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{3n_f \times 3}$, $E_i = Q_i - I_3$ and

$$\begin{aligned} h_i(v_i, g_{ij}, \tilde{E})_{j\in\mathcal{N}_i} &= \frac{k_p}{2} \sum_{j\in\mathcal{N}_i} (E_i + E_j) g_{ij}^* \\ &+ \frac{k_w}{2} \sum_{j\in\mathcal{N}_i} (E_j E_i^{\mathrm{T}} - E_i E_j^{\mathrm{T}} + E_i^{\mathrm{T}} - E_i + E_j - E_j^{\mathrm{T}}) v_i - \frac{k_v k_w}{2} \\ &\times \sum_{j\in\mathcal{N}_i} \left[\sum_{j\in\mathcal{N}_i} (E_j E_i^{\mathrm{T}} - E_i E_j^{\mathrm{T}} + E_i^{\mathrm{T}} - E_i + E_j - E_j^{\mathrm{T}}) \right] g_{ij}. \end{aligned}$$

Denote $h(v, g, \tilde{E}) = [\mathbf{0}_{n_l}, h_1^{\mathrm{T}}, \dots, h_{n_f}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{3n}$. Rewrite the closed-loop system in a compact form as

$$\begin{aligned} \dot{p} &= v, \\ \dot{v} &= M\left(k_p(g - g^*) + k_v \dot{g}\right) + h,\ \ M = -\begin{bmatrix} 0 & 0 \\ 0 & I_{3n_f} \end{bmatrix} \bar{H}^{\mathrm{T}}, \end{aligned}$$

where $g^* = [g_1^{*\mathrm{T}}, \dots, g_m^{*\mathrm{T}}]^{\mathrm{T}}$ and $\dot{g} = [\dot{g}_1^{\mathrm{T}}, \dots, \dot{g}_m^{\mathrm{T}}]^{\mathrm{T}}$.

Define the error state as $\chi := [\delta_p^{\mathrm{T}}, \delta_v^{\mathrm{T}}, \delta_g^{\mathrm{T}}]^{\mathrm{T}}$, where $\delta_p := p - p^*$, $\delta_v := v - \mathbf{1}_n \otimes v_c$ and $\delta_g := g - g^*$. The error system is expressed as a nominal system $\dot{\chi} = F(\chi, \dot{\delta}_g(t))$ with a perturbation $G(\chi, \tilde{E})$, i.e.,

$$\dot{\chi} = F(\chi, \dot{\delta}_g(t)) + G(\chi, \tilde{E}), \tag{10}$$

with

$$\begin{aligned} F(\chi, \dot{\delta}_g(t)) &= \begin{bmatrix} O_{3n} & I_{3n} & O_{3n\times 3m} \\ O_{3n} & O_{3n} & k_p M \\ O_{3n} & O_{3n} & O_{3n\times 3m} \end{bmatrix} \chi + \begin{bmatrix} \mathbf{0}_{3n} \\ k_v M \dot{\delta}_g(t) \\ \dot{\delta}_g(t) \end{bmatrix}, \\ G(\chi, \tilde{E}) &= [\mathbf{0}_{3n}^{\mathrm{T}}, h^{\mathrm{T}}, \mathbf{0}_{3m}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{6n+3m}, \end{aligned}$$

where O and $\mathbf{0}$ are both all-zero matrix and vector of suitable dimension.

In view of that the convergence of $\tilde{E}$ depends on that of $Q_i, i \in \mathcal{V}_f$, the result on the convergence rate of the proposed orientation synchronization is stated as follows.

Theorem 1: If [A1]–[A2] are satisfied, the orientation control law (8) guarantees that the orientation errors $Q_i - I_3,\ i \in \mathcal{V}_f$, asymptotically converge to a compact set containing the origin in which the relative orientations satisfy $Q_i^{\mathrm{T}} Q_j \succ 0,\ (i,j) \in \mathcal{E}$, and then locally exponentially converge to the null matrix O. ■

To prove Theorem 1, we need the following two lemmas.

Lemma 1 (Lemma 2 in [18]): The orientation matrices of followers satisfy $Q_i \succ 0,\ i \in \mathcal{V}_f$ if [A1]–[A2] are satisfied.

Lemma 2: If [A1]–[A2] are satisfied, the orientation control law (8) guarantees that the errors $Q_i - I_3,\ i \in \mathcal{V}_f$, asymptotically converge to a compact set containing the origin in which the relative orientations satisfy $Q_i^{\mathrm{T}} Q_j \succ 0,\ (i,j) \in \mathcal{E}$. ■

Proof. Assumptions [A1]–[A2] are sufficient to guarantee the asymptotic stability [18, Corollary 2], i.e., Q_i converges to I_3. Consequently, the corresponding spatial angles of the relative orientations $Q_i^{\mathrm{T}} Q_j$ converge to 0. Therefore, there exists a compact set of $Q_i - I_3$ in which the spatial angles of $Q_i^{\mathrm{T}} Q_j$ are less than $\pi/2$. The orientation errors $Q_i - I_3$ converge to this compact set where $Q_i^{\mathrm{T}} Q_j \succ 0$ holds.

Now the proof of Theorem 1 is stated as follows.

Proof of Theorem 1. Rewrite the control law (8) as

$$w_i^i = k_w \sum_{j \in \mathcal{V}_f \cap \mathcal{N}_i} sk(Q_i^{\mathrm{T}} Q_j)^{\vee} + k_w c_i sk(Q_i^{\mathrm{T}})^{\vee}, \tag{11}$$

where $c_i \in \mathbb{N}$ represents the numbers of leaders in set $\mathcal{N}_i$.

Choose a potential function $U = U_1 + U_2$ as the Lyapunov function candidate, where

$$U_1 = \frac{1}{k_w} \sum_{i=1}^{n_f} \sum_{l=1}^{n_f} \phi(Q_i^{\mathrm{T}} Q_l),\ U_2 = \frac{1}{k_w} \sum_{i=1}^{n_f} c_i \phi(Q_i^2). \tag{12}$$

Since $Q_i \in SO(3)$, we have $U = 0 \Leftrightarrow Q_i = Q_l = I_3$.

The time derivative of U_1 along the trajectory of system (9) yields

$$\begin{aligned} \dot{U}_1 &= \frac{1}{k_w} \sum_{i=1}^{n_f} \sum_{l=1}^{n_f} [sk(Q_i^{\mathrm{T}} Q_l)^{\vee}]^{\mathrm{T}} (-w_i^i + w_l^l) \\ &= PA1 + PB1, \end{aligned} \tag{13}$$

where

$$PA1 = -2 \sum_{i=1}^{n_f} \sum_{l=1}^{n_f} \sum_{j \in \mathcal{V}_f \cap \mathcal{N}_i} [sk(Q_i^{\mathrm{T}} Q_l)^{\vee}]^{\mathrm{T}} sk(Q_i^{\mathrm{T}} Q_j)^{\vee}, \tag{14}$$

$$PB1 = -2 \sum_{i=1}^{n_f} c_i \sum_{l=1}^{n_f} [sk(Q_i^{\mathrm{T}} Q_l)^{\vee}]^{\mathrm{T}} sk(Q_i^{\mathrm{T}})^{\vee}. \tag{15}$$

As (13) shows, $\dot{U}_1$ is divided into two parts, $PA1$ and $PB1$. Similarly, the time derivative of U_2 along the trajectory of system (9) leads to $\dot{U}_2 = PA2 + PB2$, where

$$PA2 = 2 \sum_{i=1}^{n_f} \sum_{j \in \mathcal{V}_f \cap \mathcal{N}_i} c_i [sk(Q_i^2)^{\vee}]^{\mathrm{T}} sk(Q_i^{\mathrm{T}} Q_j)^{\vee}, \tag{16}$$

$$PB2 = 2 \sum_{i=1}^{n_f} c_i^2 [sk(Q_i^2)^{\vee}]^{\mathrm{T}} sk(Q_i^{\mathrm{T}})^{\vee}. \tag{17}$$

It follows from [18, Theorem 2] that the following inequality holds,

$$PA1 \le -k_w \epsilon_1 \lambda_{min2}(\mathcal{L}_f) U_1, \tag{18}$$

where $\epsilon_1 = \min_{t,i,l} \lambda_{min}(\frac{Q_i^{\mathrm{T}} Q_l + Q_l^{\mathrm{T}} Q_i}{2}) > 0$ by Lemma 2.

In light of the properties of trace $x^{\mathrm{T}} y = -\frac{1}{2} tr(\hat{x}\hat{y})$, $tr(A^{\mathrm{T}}) = tr(A)$, $tr(A+B) = tr(A) + tr(B)$ and $tr(AB) = tr(BA)$, $PB1$ satisfies

$$\begin{aligned} PB1 &= \sum_{i=1}^{n_f} c_i \sum_{l=1}^{n_f} tr[sk(Q_i^{\mathrm{T}} Q_l) sk(Q_i^{\mathrm{T}})] \\ &= \frac{1}{4} \sum_{i=1}^{n_f} c_i \sum_{l=1}^{n_f} tr[(Q_i^{\mathrm{T}} Q_l - Q_l^{\mathrm{T}} Q_i)(Q_i^{\mathrm{T}} - Q_i)] \\ &= -\frac{1}{2} \sum_{i=1}^{n_f} c_i \sum_{l=1}^{n_f} tr[Q_l^{\mathrm{T}} (I_3 - Q_i^2)] \\ &\le -\frac{1}{2} \sum_{i=1}^{n_f} c_i \sum_{l=1}^{n_f} \lambda_{min}(\frac{Q_l + Q_l^{\mathrm{T}}}{2}) tr[I_3 - Q_i^2], \end{aligned} \tag{19}$$

where the last inequality follows from [21, Theorem 1]. Then, it holds that

$$PB1 \le -\frac{\epsilon_2}{2} \sum_{i=1}^{n_f} c_i \sum_{l=1}^{n_f} tr[I_3 - Q_i^2] \le 0, \tag{20}$$

where $\epsilon_2 = \min_{t,l} \lambda_{min}(\frac{Q_l + Q_l^{\mathrm{T}}}{2}) = \min_{t,i} \lambda_{min}(\frac{Q_i + Q_i^{\mathrm{T}}}{2}) = \min_{t,j} \lambda_{min}(\frac{Q_j + Q_j^{\mathrm{T}}}{2}) > 0$ by Lemma 1.

Similarly, with $c_i^2 \ge c_i$, we have the following inequalities,

$$PA2 \le -\frac{\epsilon_2^2}{2} \sum_{i=1}^{n_f} c_i \sum_{j \in \mathcal{V}_f \cap \mathcal{N}_i} tr[I_3 - Q_i^2] \le 0, \tag{21}$$

$$PB2 \le -\epsilon_2 \sum_{i=1}^{n_f} c_i \phi(Q_i^2) = -k_w \epsilon_2 U_2. \tag{22}$$

It follows from (18), (20), (21), and (22) that

$$\begin{aligned} \dot{U} = \dot{U}_1 + \dot{U}_2 &\le -k_w \epsilon_1 \lambda_{min2}(\mathcal{L}_f) U_1 - k_w \epsilon_2 U_2 \\ &\le -\epsilon(U_1 + U_2) = -\epsilon U, \end{aligned} \tag{23}$$

where $\epsilon = \min(k_w \epsilon_1 \lambda_{min2}(\mathcal{L}_f), k_w \epsilon_2) > 0$. By the comparison principle [22, Lemma 3.4], we have

$$U \le U(t_0) e^{-\epsilon(t - t_0)}. \tag{24}$$

As a result, $\{Q_i = I_3\},\ i \in \mathcal{V}_f$ is the locally exponentially stable equilibrium point set, that is, $Q_i \to I_3$ exponentially as $t \to \infty$. A similar proof of local orientations' exponential convergence was made in [18]. The proof is completed.

Now, we present the main result as follows.

Theorem 2: (Semi-Globally Asymptotic Stability) Under the control laws (7) and (8), the error state χ of system (10) converges to 0 if [A1]–[A4] are satisfied. ■

Proof of Theorem 2. A technical lemma is given in the Appendix to prove the semi-global stability. Firstly, consider the nominal system $\dot{\chi} = F(\chi, \dot{\delta}_g(t))$ with $\chi(t_0) \in \mathcal{X}$, where the domain $\mathcal{X}$ is defined as $\mathcal{X} := \{\chi : \|\delta_p\| \le B_p, \quad \|\delta_v\| \le B_v\}$ since $[p^{\mathrm{T}}(t_0), v^{\mathrm{T}}(t_0)]^{\mathrm{T}} \in \Xi_0$ by [A3].

Consider a Lyapunov function candidate $V : \mathbb{R}_{\ge 0} \times \mathcal{X} \to \mathbb{R}_{\ge 0}$ as $V(t, \chi) = k_p e^{\mathrm{T}}(g - g^*) + \frac{1}{2}\delta_v^{\mathrm{T}} \delta_v = k_p \delta_p^{\mathrm{T}} \bar{H}^{\mathrm{T}} \delta_g + k_p e^{*\mathrm{T}} \delta_g + \frac{1}{2} \delta_v^{\mathrm{T}} \delta_v$, where $e^* = \bar{H} p^*(t)$ is time-invariant in Definition 1. By [16, Lemma 2], $V(t, \chi)$ is positive definite, and by [22, Lemma 4.3], subcondition (i) of [C1] in Lemma 3 is satisfied.

Taking the time derivative of $V(t, \chi)$ along the trajectory of system $\dot{\chi} = F(\chi, \dot{\delta}_g(t))$ yields $\dot{V}(t, \chi) = -k_v \sum_{k=1}^{m} \dot{e}_k^{\mathrm{T}} \frac{P_{g_k}}{\|e_k\|} \dot{e}_k$, which is negative semi-definite by [16, Theorem 3]. Thus, the nominal system $\dot{\chi} = F(\chi, \dot{\delta}_g(t))$ is uniformly asymptotically stable in the domain $\mathcal{X}$, and subcondition (ii) of [C1] in Lemma 3 is satisfied.

Next, for all $\chi \in \mathcal{X}$, we have

$$\|\chi\| = \sqrt{\|\delta_p\| + \|\delta_v\| + \|\delta_g\|} \leq \sqrt{B_p^2 + B_v^2 + 4m} := \iota_1,$$

since $\|\delta_g\| \leq \|g\| + \|g^*\| = \sqrt{\|g_1\|^2 + \ldots + \|g_m\|^2} + \sqrt{\|g_1^*\|^2 + \ldots + \|g_m^*\|^2} = 2\sqrt{m}$. Obviously, there exists a positive constant σ such that

$$\max(\sqrt{B_p^2 + 4m + \sigma}, \sqrt{B_v^2 + 4m + \sigma}) < \iota_1.$$

Choose a positive constant ζ such that

$$\max(\sqrt{B_p^2 + 4m + \sigma}, \sqrt{B_v^2 + 4m + \sigma}) < \zeta < \iota_1.$$

Without loss of generality, suppose $B_p > B_v$. It follows that

$$\max(\sqrt{B_p^2 + 4m + \sigma}, \sqrt{B_v^2 + 4m + \sigma}) = \sqrt{B_p^2 + 4m + \sigma}.$$

For any $\chi \in \mathcal{X}$ such that $0 < \sqrt{B_p^2 + 4m + \sigma} < \zeta \leq \|\chi\| \leq \iota_1$, we have $\|\delta_p\|^2 + \|\delta_v\|^2 + \|\delta_g\|^2 = \|\chi\|^2 > B_p^2 + 4 + \sigma$, i.e., $\|\delta_v\|^2 > (B_p^2 - \|\delta_p\|^2) + (4m - \|\delta_g\|^2) + \sigma \geq \sigma > 0$. As a result, $V(t, \chi)$ satisfies $V(t, \chi) \geq \frac{1}{2}\|\delta_v\|^2 > \frac{1}{2}\sigma$. Compute the norm of $\frac{\partial V}{\partial \chi} = [k_p \delta_g^{\mathrm{T}} \bar{H}, \delta_v^{\mathrm{T}}, k_p(\delta_p^{\mathrm{T}} \bar{H}^{\mathrm{T}} + e^{*\mathrm{T}})]$ as

$$\left\|\frac{\partial V}{\partial \chi}\right\| = \sqrt{k_p^2 \|\bar{H}^{\mathrm{T}} \delta_g\|^2 + \|\delta_v\|^2 + k_p^2 \|\bar{H}\delta_p + e^*\|^2}$$
$$\leq \sqrt{4mk_p^2 \|\bar{H}\|^2 + B_v^2 + k_p^2 (B_p \|\bar{H}\| + \|e^*\|)^2} := \iota_2, \quad (25)$$

and it follows that $\left\|\frac{\partial V}{\partial \chi}\right\| \|\chi\| \leq \iota_1 \iota_2$. Choose $b_1 \geq \frac{2\iota_1 \iota_2}{\sigma}$, then subcondition (iii) of [C1] in Lemma 3 is satisfied.

From (25), choose $b_2 \geq \iota_2 > 0$, then subcondition (iv) of [C1] in Lemma 3 holds. Thus, [C1] in Lemma 3 is satisfied.

Secondly, define $\mathcal{Y} = \{\tilde{E} = [Q_1^{\mathrm{T}} - I_3, \ldots, Q_{n_f}^{\mathrm{T}} - I_3]^{\mathrm{T}}\}$, where $Q_i \succ 0$ for all $i \in \mathcal{V}_f$. By Theorem 1, $\tilde{E} = O$ is asymptotically stable in $\mathcal{Y}$. Moreover, for any $\tilde{E} \in \mathcal{Y}$, there exists a compact set $\bar{\mathcal{Y}} \subset \mathcal{Y}$ with an instant $t_1 > 0$ such that $\tilde{E} \to 0$ exponentially in $\bar{\mathcal{Y}}$ after the instant t_1. Thus [C2] in Lemma 3 is satisfied.

Thirdly, since $\|E_i\| \leq \|\tilde{E}\|$, $\|E_j\| \leq \|\tilde{E}\|$, $\|v_i\| \leq \|\delta_{v_i}\| + v_c$, $\|g_{ij}\| = 1$, $\|\delta_{v_i}\| \leq \|\delta_v\| \leq \|\chi\|$ and $|\mathcal{N}_i| \leq n$, we have

$$\|G(\chi, \tilde{E})\| = \|h(v, g, \tilde{E})\| \leq \sum_{i=1}^{n_f} \|h_i(v_i, g_{ij}, \tilde{E})_{j \in \mathcal{N}_i}\|$$
$$\leq \sum_{i=1}^{n_f} [k_p |\mathcal{N}_i| \|\tilde{E}\| + k_w |\mathcal{N}_i| \|\tilde{E}\| (\|\tilde{E}\| + 2)(\|\delta_{v_i}\| + \|v_c\|)$$
$$+ k_v k_w |\mathcal{N}_i|^2 \|\tilde{E}\| (\|\tilde{E}\| + 2)]$$
$$\leq \|\tilde{E}\| [\Theta_1(\|\tilde{E}\|) + \|\chi\| \Theta_2(\|\tilde{E}\|)],$$

with $\Theta_1(\|\tilde{E}\|) = n^2 \left(k_p + k_w(\|\tilde{E}\| + 2)(\|v_c\| + nk_v)\right)$ and $\Theta_2(\|\tilde{E}\|) = (n^2 k_w (\|\tilde{E}\| + 2))$. Then [C3] in Lemma 3 holds.

By Lemma 3, system (10) is uniformly asymptotically stable for $\chi \in \mathcal{X}$. The proof is completed.

IV. AN ILLUSTRATIVE EXAMPLE

In this section, an example is given to illustrate control laws (7) and (8) designed for double-integrator followers. The target formation is a cube, consisting of two leaders, $\mathcal{V}_l = \{1, 2\}$ and six followers $\mathcal{V}_f = \{3, \ldots, 8\}$. The cube is defined by the bearing constraints $g_{4,1}^* = g_{3,2}^* = -g_{5,6}^* = g_{7,8}^* = [0, 1, 0]^{\mathrm{T}}$, $g_{4,3}^* = -g_{2,1}^* = g_{5,8}^* = g_{6,7}^* = [1, 0, 0]^{\mathrm{T}}$, $g_{4,6}^* = g_{3,7}^* = g_{2,8}^* = -g_{5,1}^* = [0, 0, -1]^{\mathrm{T}}$ and $g_{4,8}^* = \frac{1}{\sqrt{3}}[1, 1, -1]^{\mathrm{T}}$ in the leaders' reference frame Σ_l. The initial positions, velocities and orientations' rotation angles are listed in Table I. Set the control gains $k_p = 0.5$, $k_v = 4.6$ and $k_w = 0.5$. As is shown in Fig. 1, the formation converges to the desired cube. Fig. 2 shows that the velocities of followers converge to those of the leaders. Figs. 3 and 4 illustrate that the orientation errors and bearing error converge to zero.

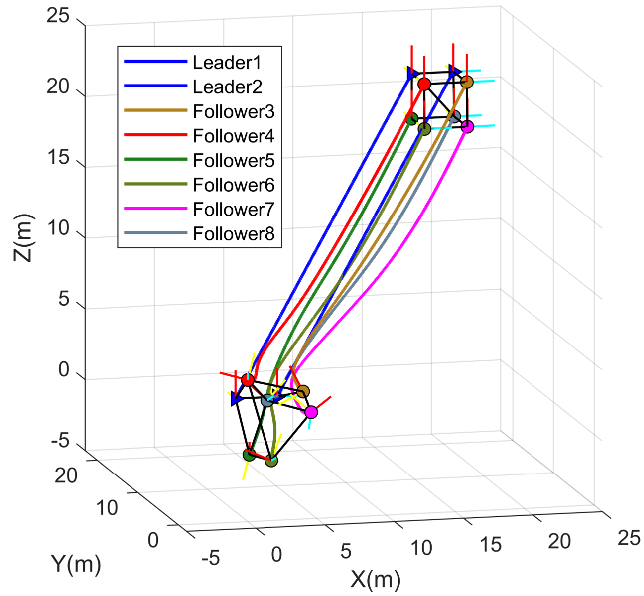

Fig. 1. The trajectories of leaders and followers.

TABLE I
INITIAL POSITIONS, VELOCITIES AND ROTATION ANGLES

i	$p_i(0)(m)$	$v_i(0)(m \cdot s^{-1})$	$\alpha_i(0)$	$\beta_i(0)$	$\gamma_i(0)$
1	$[0, 3, 3]^{\mathrm{T}}$	$[0.30, 0.30, 0.30]^{\mathrm{T}}$	0	0	0
2	$[3, 3, 3]^{\mathrm{T}}$	$[0.30, 0.30, 0.30]^{\mathrm{T}}$	0	0	0
3	$[4, 0, 4]^{\mathrm{T}}$	$[-0.22, 0.47, -0.02]^{\mathrm{T}}$	$-\pi/2.5$	$\pi/6$	$\pi/10$
4	$[0, 0, 5]^{\mathrm{T}}$	$[-0.24, 0.02, 0.46]^{\mathrm{T}}$	$\pi/4$	$-\pi/3$	$-\pi/2.8$
5	$[1, 3, -1]^{\mathrm{T}}$	$[0.32, 0.29, -0.29]^{\mathrm{T}}$	$-\pi/6$	$\pi/2.5$	$\pi/4$
6	$[2, 1, -1]^{\mathrm{T}}$	$[-0.17, -0.14, 0.47]^{\mathrm{T}}$	$\pi/3$	$-\pi/3$	$-\pi/4.5$
7	$[4, -2, 3]^{\mathrm{T}}$	$[0.11, 0.50, 0.05]^{\mathrm{T}}$	$-\pi/4.5$	$-\pi/6$	$\pi/3$
8	$[2, 2, 3]^{\mathrm{T}}$	$[0, -0.11, 0.51]^{\mathrm{T}}$	$\pi/2.5$	$-\pi/4.5$	$-\pi/6$

V. CONCLUSIONS

In this paper, a dynamic bearing-only control law has been proposed to solve the leader-following formation tracking problem of double-integrators in local reference frames. The underlying topology among agents is described by a multiply rooted undirected graph. Only relative bearings and relative orientations measured in each agent's body-fixed local reference frame are utilized to design the control law. In the future, the translational, scaling and affine formation maneuver problems where the leaders' velocities are time-varying will be studied, and the condition for collision avoidance will also be investigated.

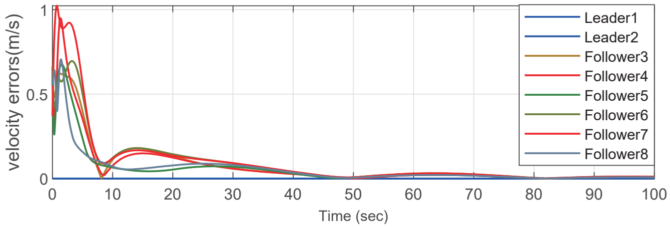

Fig. 2. The velocity errors $\|v_i - v_c\| \quad (i \in \mathcal{V})$

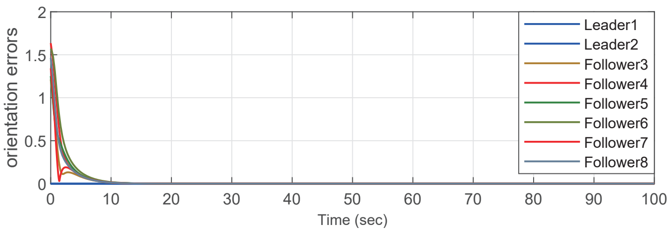

Fig. 3. The orientation errors $\|Q_i - I_3\| \quad (i \in \mathcal{V})$

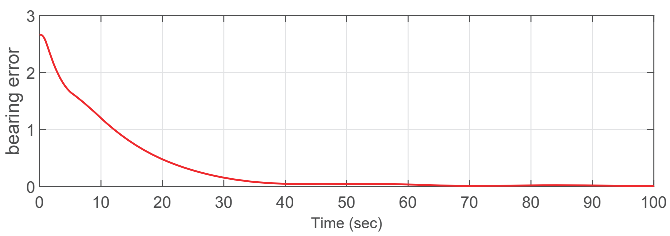

Fig. 4. The bearing error $\sum_{(i,j)\in\mathcal{E}} \|g_{ij} - g_{ij}^*\|$

APPENDIX

The following lemma on the perturbed nonlinear system directly follows from [19, Lemma 2.1].

Lemma 3: Consider the following system:

$$\dot{\chi} = F(\chi, \eta(t)) + G(\chi, \tilde{E}), \tag{26}$$

where $\chi \in \mathbb{R}^{n_s}$ is the state, $\tilde{E} \in \mathbb{R}^{n_{e1}\times n_{e2}}$ is an exogenous signal, $\eta(t)$ is a time-varying function in a compact set. $F(\chi, \eta(t))$ and $G(\chi, \tilde{E})$ are continuous in their arguments. $F(\chi, \eta(t))$ is locally Lipschitz on χ uniformly on η. $G(\chi, \tilde{E})$ is locally Lipschitz on $(\chi, \tilde{E})$. System (26) can be considered as a perturbation of the nominal system $\dot{\chi} = F(\chi, \eta(t))$.

Let $\chi = 0$ be an equilibrium point for system (26) and $\mathcal{X} \subset \mathbb{R}^{n_s}$ be a domain containing $\chi = 0$. System (26) is uniformly asymptotically stable for $\chi \in \mathcal{X}$ if the following conditions [C1]–[C3] are satisfied.

[C1] The nominal system $\dot{\chi} = F(\chi, \eta(t))$ is uniformly asymptotically stable for $\chi \in \mathcal{X}$ with a Lyapunov function $V : \mathbb{R}_{\geq 0} \times \mathcal{X} \mapsto \mathbb{R}_{\geq 0}$, such that the following subconditions holds: (i) $\underline{W}(\chi) \leq V(t,\chi) \leq \overline{W}(\chi)$; (ii) $\frac{\partial V(t,\chi)}{\partial t} + \frac{\partial V(t,\chi)}{\partial \chi} F(\chi, \eta(t)) \leq -W(\chi)$; (iii) $\left\| \frac{\partial V(t,\chi)}{\partial \chi} \right\| \|\chi\| \leq b_1 V(t,\chi),\ \forall \|\chi\| \geq \zeta$; (iv) $\left\| \frac{\partial V(t,\chi)}{\partial \chi} \right\| \leq b_2,\ \forall \|\chi\| \leq \zeta$, where $\underline{W}(\chi)$ and $\overline{W}(\chi)$ are two class $\mathcal{K}$ functions, $W(\chi)$ is a positive semi-definite function, and $b_1 > 0$, $\zeta > 0$ and $b_2 > 0$ are some constants.

[C2] There exists a class $\mathcal{KL}$ function $\varphi(\cdot)$ such that for all $t \geq t_0 \geq 0$ and $\tilde{E}(t_0) \in \mathcal{Y} \subset \mathbb{R}^{n_{e1}\times n_{e2}}$, $\|\tilde{E}(t)\| \leq \varphi(\|\tilde{E}(t_0)\|, t - t_0)$, and a compact set $\bar{\mathcal{Y}} \subset \mathcal{Y}$ with time $t_1 > t_0 \geq 0$ and two positive constants k and ς, such that for all $t \geq t_1 > t_0 \geq 0$ and $\tilde{E}(t_1) \in \bar{\mathcal{Y}}$, $\|\tilde{E}(t)\| \leq k\|\tilde{E}(t_1)\| e^{-\varsigma(t-t_1)}$.

[C3] The function $G(\chi, \tilde{E})$ satisfies that for $\chi \in \mathcal{X}$ and $\tilde{E} \in \mathcal{Y}$, $\|G(\chi, \tilde{E})\| \leq \|\tilde{E}\| \left(\Theta_1(\|\tilde{E}\|) + \|\chi\|\Theta_2(\|\tilde{E}\|)\right)$, where $\Theta_1 : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ and $\Theta_2 : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ are continuous functions. ■

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